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B.E. / B.Tech (Full Time) DEGREE EXAMINATIONS APRIL/MAY 2012

40

SIXTH SEMESTER

CS 9032 – GRAPH THEORY

REGULATIONS – 2008

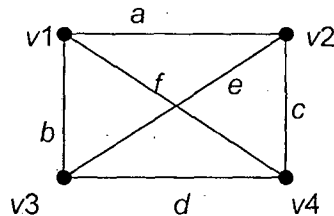
Time : 3 hours

Max. Marks : 100

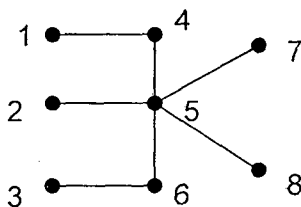
Answer ALL Questions

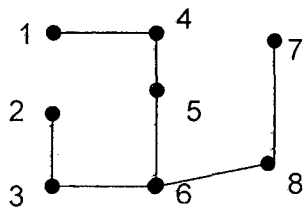
PART A – (10 x 2 = 20 marks)

1. Which of the following are true with respect to the following graph G .

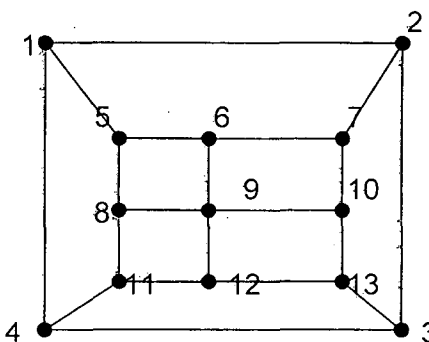
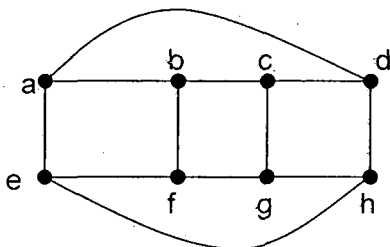


- (i) G is a subgraph of G
 - (ii) Vertex v_2 is a subgraph of G
 - (iii) ecd and ab are vertex disjoint subgraphs of G
 - (iv) cd is a subgraph of ecd
2. Prove that in a connected graph with exactly $2k$ odd vertices, there exist k edge-disjoint subgraphs such that they contain all edges of G and that each is a unicursal graph.
3. Transform the first graph to the second given below the first by a sequence of trees each of which differs from the next by a single edge. Find the distance between the initial tree and the final tree.





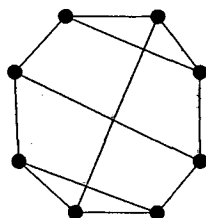
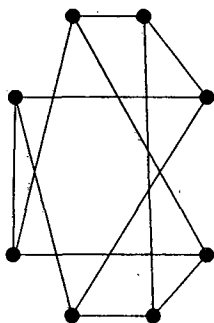
4. When are two graphs said to have circuit correspondence?
5. In a path matrix, what can you say about a column of all 0s, a column of all 1s?
6. Determine whether the following graphs are bipartite. If so, give the partition of the vertices into two sets.



7. Write in brief about any two ways in which a graph can be stored in a computer.
8. When repeated fusion of adjacent vertices is done to find whether a graph is connected or not, what will be the maximum number of fusions that need to be done when there are n vertices in the graph?
9. What is called shortest distance arborescence?
10. What is meant by an invariant of a graph? Give any two examples of invariants of a graph.

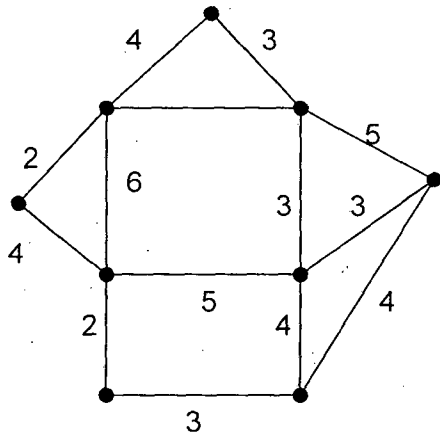
PART B – (5 x 16 = 80 marks)

11. a) If you flip a coin three times, there are 8 ways the coins can come up: HHH, HHT, HTH, etc.... Suppose these are the labels on 8 vertices of a graph, and that you draw an edge between a pair of vertices if the labels differ in exactly one place. For example, HTH will be adjacent to HTT, but will not be adjacent to THH. Find if the resulting graph is isomorphic to any or all of the following graphs. (8)



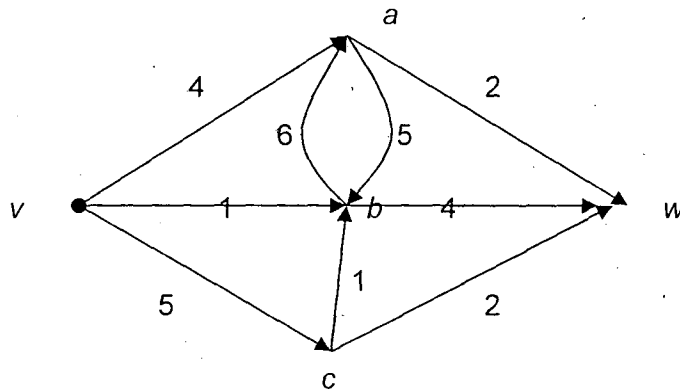
- b) Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)$ edges. (5)
- c) Sketch all binary trees with six pendant vertices. (3)

- 12 a) (i) Prove that the nullity of a graph does not change when you either insert a vertex in the middle of an edge, or remove a vertex of degree two by merging two edges incident on it. (8)
- (ii) Apply Prim's algorithm to find a minimal spanning tree for the graph given below. Explain the procedure step by step. (8)



(OR)

- 12 b) (i) List all the cut-sets in the following network, with respect to the vertices v and w , and find the cut-set with the minimum capacity. Find the maximum flow possible between v and w and verify that this satisfies the max-flow min-cut theorem. (8)

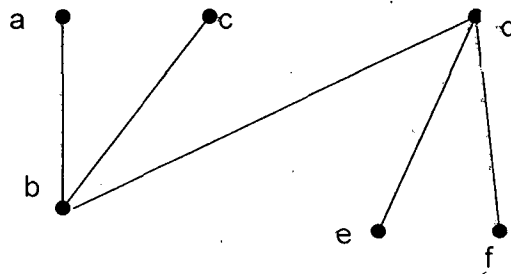


- (ii) Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ regions. (8)

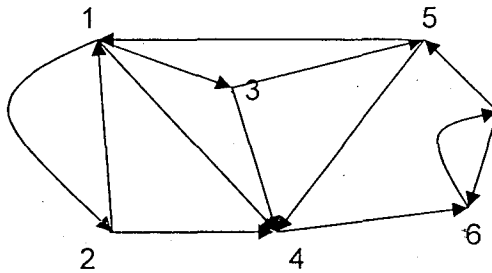
13. a) (i) For an adjacency matrix X , show with an example that each diagonal entry in X^3 equals twice the number of triangles passing through the corresponding vertex. (10)
 (ii) Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. (6)

(OR)

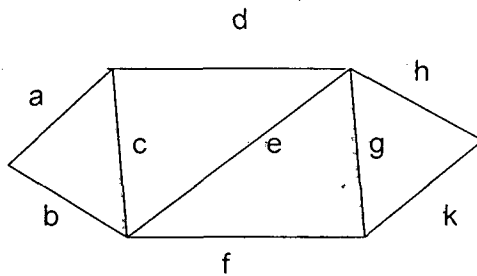
13. b) (i) Find all the maximal independent sets for the following graph using Boolean arithmetic: (10)



- (ii) Find the condensation of the graph given below: (6)

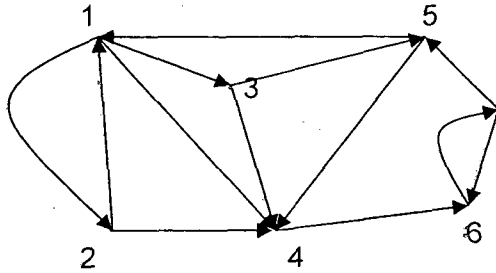


14. a) Use Paton's algorithm to find all the fundamental circuits in the following graph. Show the step-by-step procedure. (16)

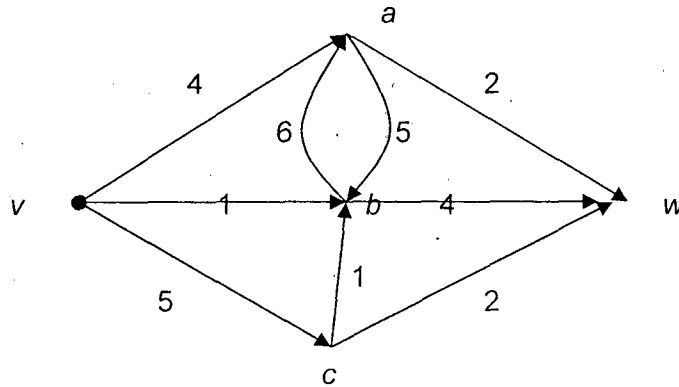


(OR)

14. b) Use Roberts and Flores' algorithm to find all the directed circuits in the following graph. Show the step-by-step procedure. (16)



15. a) Use Dijkstra's algorithm to find the shortest path from vertex v to vertex w in the following graph. Explain the step-by-step procedure. (16)



(OR)

15. b) Explain the planarity testing algorithm proposed by Hopcroft and Tarjan. Discuss about the technique they have used to resolve ambiguity in adding paths. (16)