Reg. No.										

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B. E/ B. Tech.(Full Time) DEGREE END SEMESTER EXAMINATIONS , MAY 2012

COMPUTER SCIENCE AND ENGINEERING

FOURTH SEMESTER

MA 9265 – DISCRETE MATHEMATICS

(REGULATION 2008)

Time : 3hr **Max Mark : 100**

Answer ALL Questions

Part – A (10 x 2 = 20 Marks)

1. Obtain the contrapositive and the inverse of the statement given below. " *If tomorrow is a holiday then I will go for a trip.*"

2. What are the negations of the statements $\forall x(x^2 > x)$ and $\exists x(x^2 = 2)$?

3. Find the number of arrangements of the letters *SUCCESS*. How many of these arrangments have no adjacent *S* ?

4. Show that the sum of the first n odd positive integers is n^2.

5. Obtain the incidence matrix of the graph given in **Figure Q5.** What is the row sum corresponding to a vertex in the inciidence matrix.

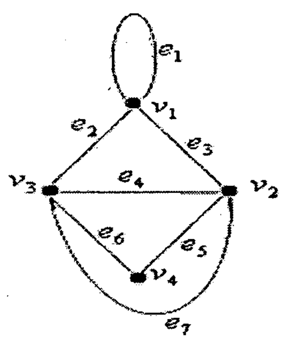


Figure Q5

6. Give an example of an Eulerian graph which is non-Hamiltonian and not bipartite.

7. Let $(G, *)$ be a group. For $a, b \in G$, if $(a*b)^2 = a^2 * b^2$ then prove that $(G, *)$ is abelian.

8. Give an example of a commutative ring with unity and having zero divisors. Check whether your example is an integral domain.

9. Show that in a lattice if $a \leq b \leq c$ then $(a * b) \oplus (b * c) = (a \oplus b) * (a \oplus c)$.

10. Give an example of a lattice which is complemented but not distributive. Justify your answer.

Part – B (5 x16 = 80 Marks)

11. (i) Determine the number of postive integer $n, 1 \leq n \leq 250$ that are not divisible by 2, 3, or 5. (8 Marks)

(ii) Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$, where $n \geq 2$, $a_1 = 5$ and $a_2 = 3$. (8 Marks)

12. a(i) Show that $(\neg P \to R) \wedge (P \to Q) \wedge (Q \to P)$

$$\equiv (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R).$$

(6 Marks)

(ii). Prove that $\exists x(P(x) \wedge Q(x)) \Rightarrow \exists x(P(x)) \wedge \exists x(Q(x))$. Check whether its converse is true. Justify your answer. (10 Marks)

OR

b(i) Show that premises, " *A student in this class has not read the book* ", and "*Everyone in the class passed the semester exam* ", imply the conclusion, " *Someone who passed the semester exam has not read the book* ". (10 Marks)

(ii) Prove the statement, " If $3n+2$ is odd then n is odd, by giving a proof by contradiction. (6 Marks)

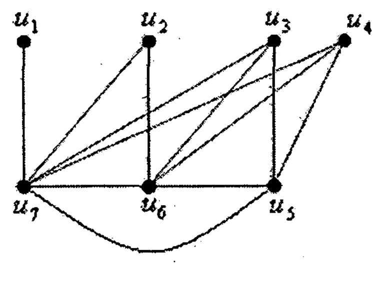
13. (a)(i) If *G* is a simple graph with *n* vertices with $n \geq 3$ such that degree of every vertex is at least $\frac{n}{2}$, then prove that *G* has Hamiltonian cycle. Check whether the converse is true. Justify your answer. (10 Marks)

(ii) When do we say a simple graph is self-complementary. If *G* is self-complementary graph with *n* vertices then prove that $n \equiv 0 \text{ or } 1 \pmod 4$.

(6 Marks)

OR

(b)(i) Check whether the graphs given in **Figure Q 13(b)(i)** are isomorphic. Justify your answer.



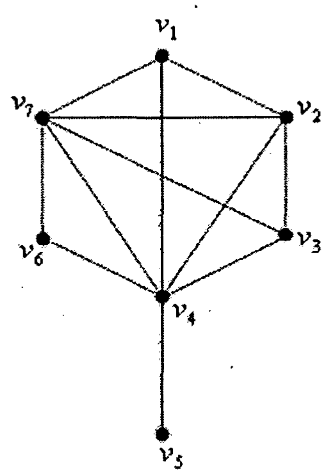


Figure Q13(b)(i)

(8 Marks)

(ii) Prove the following:

(1) A (simple) graph *G* is bipartite if and only if the vertices of *G* can be coloured with two colours such a way that the adjacent vertices always receive different colours.

(2) Complete bipartite graph $K_{m,n}$ is Hamiltonian if and only if $m = n$.

(8 Marks)

14(a)(i) Let $(G, *)$ be a finite cyclic group of order *n* and generated by an element $a \in G$. Then prove that $G = \{a, a^2, \cdots, a^n = e\}$ and *n* is the least positive integer such that $a^n = e$. (8 Marks)

(ii) Let $(G, *)$ be a group and let *H* be a normal subgroup of *G*. Then prove that $(G|H, \otimes)$ is a group, where $G|H = \{aH \mid a \in G\}$ and for $aH, bH \in G|H$, $aH \otimes bH = (a*b)H$. Further, prove that there exists a natural homomorphism *g* from $(G, *)$ onto $(G|H, \otimes)$. (8 Marks)

OR

(b)(i) Obtain all the elements of (S_3, ◊) and construct its composition table. Check whether the subgroup $\left\{\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right\}$ is a normal subgroup of (S_3, ◊). (12 Marks)

(ii) If $(G,*)$ is a finite group of order *n*, then prove that for any $a \in G$, $a^n = e$, where e is the identity element of *G*. (4 Marks)

15(a) Show that in a lattice *L*, the following are true.

(i) $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

(ii) $b \le c \Rightarrow \begin{cases} a * b \le a * c \\ a \oplus b \le a \oplus c \end{cases}$

(iii) $a * (b \oplus c) \ge (a * b) \oplus (a * c)$ and $a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$.

(16 Marks)

OR

(b) Show that in a complemented and distributive lattice *L* the following are true.

(i) Complement of every element in *L* is unique.

(ii) $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$, $\forall\ a, b \in L$.

(iii) $a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \le a'\ \ \forall\ a, b \in L$.

(16 Marks)

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