B.E / B.TECH (FULL TIME) DEGREE END SEMESTER EXAMINATIONS, APR-MAY 2012
MECHANICAL ENGINEERING BRANCH
VI SEMESTER (REGULATION 2008)
(COMMON TO MECH, MANUF & MATERIAL SCIENCE)
ME 9351 – FINITE ELEMENT ANALYSIS

Time: 3 Hours — Max. Marks: 100

PART-A
(10 x 2 = 20 Marks)

1. What are the advantages of weak formulation?
2. Derive the governing equation for one dimensional heat transfer through a fin and state the boundary conditions?
3. What are the properties of shape functions? What is the significance of shape function?
4. Derive M_{13} term of the mass matrix for a beam element
5. What does $\iint N_i \, dx \, dy$ yield for a constant strain triangular element?.
6. Derive the shape function for the internal node of a quadratic quadrilateral Lagrangean element.
7. Derive the constitutive matrix for plane stress element.
8. Give the B (Strain displacement) matrix for a linear quadrilateral element.
9. What are natural coordinate systems? What are the advantages of the same?
10. Derive the shape functions of an ID quadratic iso parametric element.

PART-B
(5 x 16 = 80 Marks)

11. For the tapered steel bar shown in Fig 11, subjected to its own self weight, determine the deflection at the free end using any weighted residual technique or the Ritz technique. Assume E=200GPa and γ=77kN/m^2

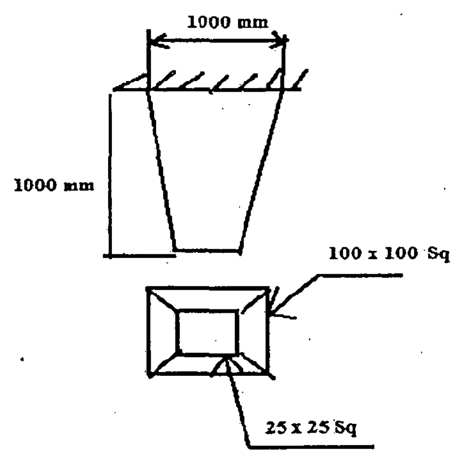


Fig 11

12. a) Calculate the temperature distribution in the fin of constant cross section shown in Fig 12a. Assume that the free end is open to the atmosphere and that the thermal conductivity is 0.2 W/cm°C and the convective coefficient is 0.0025W/cm^2°C. The ambient temperature is 25°C

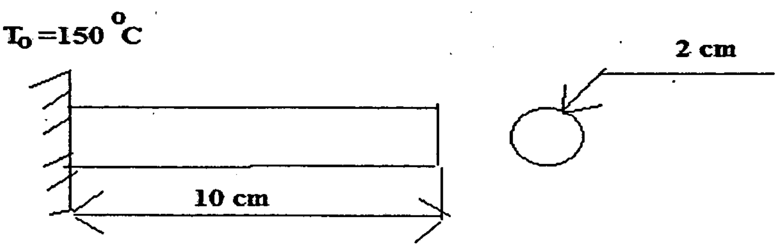


Fig 12a

(OR)

12\. b) Determine the first two natural frequencies of longitudinal vibration of the steel stepped bar shown in Fig .12b and plot the mode shapes. All dimensions are in mm.

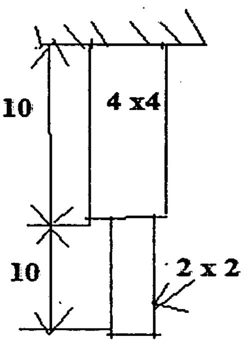
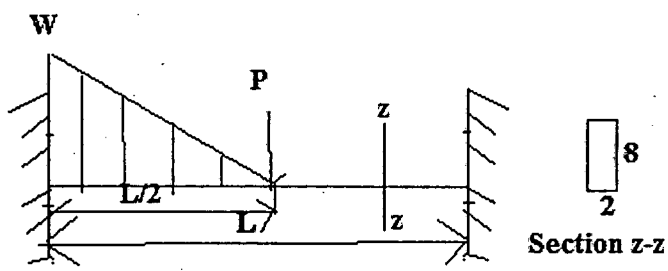


Fig 12b

13\. a) Determine the max deflection and reaction in the fixed fixed beam shown in Fig 13a. How will you compute the maximum stress? W=2kN; P=10kN; L=200cm.



(OR)

13\. b) (i)Determine the first two natural frequencies of the transverse vibration of a cantilever beam (12 Marks)
(ii) Differentiate between consistent and lumped mass matrix.(4 Marks)

14\. a) (i)Determine three points on the 56° contour line for a rectangular element shown in Fig 14a T_1=40° , T_2=65° , T_3=60° and T_4=52°(10 Marks)
(ii) Derive the K_{11} and K_{32} terms of the stiffness matrix for a four noded bilinear rectangular element used in scalar variablel problem (6 marks)

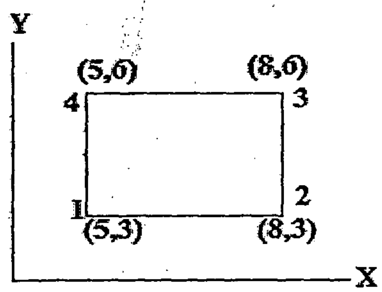


Fig 14a

(OR)

14. b) (i)Determine the strain displacement matrix, Constitutive matrix and nodal force vector for the element as shown in fig 14b. How is the stiffness matrix derived (12 Marks)

(ii) Derive the stiffness matrix for a one dimensional axial element using strain energy approach (4 Marks)

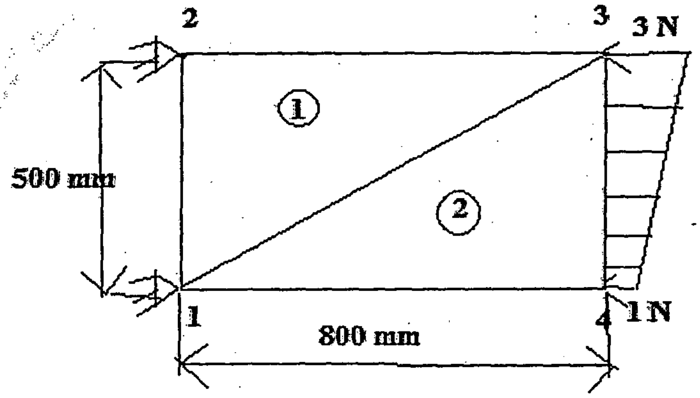


Fig 14b

15.a) (i)For the four noded element shown in Fig 15a determine the Jacobian and evaluate its value at the point (1/4, 1/4) (8 Marks)

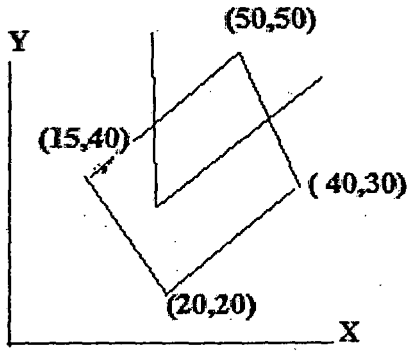


Fig 15a

(ii) Clearly bring out the difference between sub, iso and super parametric element. (8 Marks)

(OR)

15.b) (i)Evaluate the integral of the function using Gaussian intergration.Compare with exact values (6 Marks)

$I=\int(1+\varepsilon+\varepsilon^2+\varepsilon^3)d\varepsilon$

(ii) What is the concept behind Gauss intergration method? Why should the weights add to 2? (2 Marks)

(ii) Derive the shape function for one corner node and one mid side node of a quadratic quadrilateral serendipity element.(8 Marks)

$$K=\frac{EI}{L^3}\begin{bmatrix}12 & 6L & -12 & 6L\\ 6L & 4L^2 & -6L & 2L^2\\ -12 & -6L & 12 & -6L\\ 6L & 2L^2 & -6L & 4L^2\end{bmatrix}$$

$$M=\frac{\rho AL}{420}\begin{bmatrix}156 & 22L & 54 & -13L\\ 22L & 4L^2 & 13L & -3L^2\\ 54 & 13L & 156 & -22L\\ -13L & -3L^2 & -22L & 4L^2\end{bmatrix}$$

$$\{F\}=\frac{fL}{12}\begin{Bmatrix}6\\ L\\ 6\\ -L\end{Bmatrix}$$

$$N_1(x)=1-3x^2/L^2+2x^3/L^3$$
$$N_2(x)=x-2x^2/L+x^3/L^2$$
$$N_3(x)=3x^2/L^2-2x^3/L^3$$
$$N_4(x)=-x^2/L+x^3/L^2$$

Table 4.1. Locations (r_i) and Weights (w_i) in Gaussian Integration (Eq. 4.121)

Number of points (n)	Location (r_i)	Weight (w_i)
1	r_1 = 0.00000 00000 00000	2.00000 00000 00000
2	r_1, r_2 = ±0.57735 02691 89626	1.00000 00000 00000
3	r_1, r_3 = ±0.77459 66692 41483	0.55555 55555 55555
	r_2 = 0.00000 00000 00000	0.88888 88888 88889
4	r_1, r_4 = ±0.86113 63115 94053	0.34785 48451 47454
	r_2, r_3 = ±0.33998 10435 84856	0.65214 51548 62546
5	r_1, r_5 = ±0.90617 98459 38664	0.23692 68850 56189
	r_2, r_4 = ±0.53846 93101 05683	0.47862 86704 99366
	r_3 = 0.00000 00000 00000	0.56888 88888 88889
6	r_1, r_6 = ±0.93246 95142 03152	0.17132 44923 79170
	r_2, r_5 = ±0.66120 93864 66265	0.36076 15730 48139
	r_3, r_4 = ±0.23861 91860 83197	0.46791 39345 72691