Reg. No. | | | | | | | | | |

END SEMESTER EXAMINATIONS – APRIL 2011
FOURTH SEMESTER
B.E. / B.TECH
MA9261 PROBABILITY AND STATISTICS

[Common to Geo-Informatics, Manufacturing Engineering, Industrial Engineering, Textile Technology, Industrial Bio-technology, Food Technology, Pharmaceutical Technology and Apparel Technology]

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Use of Statistical Tables is permitted

Time: 3 Hours | **Answer ALL Questions** | **Max. Marks: 100**

Part A **(10 × 2 = 20)**

1. The p.d.f of the random variable X is given by $f(x)=\frac{k}{x^2+1}$, $-\infty<x<\infty$ .Find the value of k .

2. If random variable X is uniformly distributed in (- 3 , 3), find the probability density function of Y= 4 X + 3.

3. The two lines of regression are 3x + 12y – 19 = 0, 9x + 3y – 46 = 0. Find the value of correlation coefficient between x and y.

4. A random sample of size 100 is taken from an infinite population having the mean $\mu = 76$ and the variance of $\sigma^2 = 256$. What is the probability that $\overline{X}$ will be between 75 and 78.

5. An oil company claims that less than 20 percent of all car owners have not tried its gasoline. Test this claim at the 0.01 level of significance if a random check reveals that 22 of 200 car owners have not tried the oil company's gasoline.

6. If 12 determinations of specific heat of iron have a standard deviation of 0.0086, test the null hypothesis that σ = 0.0100 for such determinations. Use the alternative hypothesis $\sigma \neq$ 0.0100 at the 0.01 level of significance.

7. State the identity for sum of squares for one - way of analysis of variance.

8. What is the Latin Square Design.

9. A garment was sampled on 10 consecutive hours of production. The number of defects found per garment is given below:

   Defects : 5,1,7,0,2,3,4,0,3,2 . Compute upper and lower control limits for monitoring number of defects.

10. To check the strength of carbon steel for use in chain links, the yield stress of a random sample of 25 pieces was measured , yielding a mean and standard deviation of 52,800 psi and 4600 psi, respectively. Establish tolerance limits with $\alpha = 0.05$ and $P = 0.99$.

Part B **(5 × 16 = 80)**

11. (i) The following data gives the average life in hours and range in hours of 12 samples each of 5 lamps .Construct $\overline{X}$ - chart and R-chart, comment on state of control. (10)

Sample No	1	2	3	4	5	6	7	8	9	10	11	12
$\overline{X}$	120	127	152	157	160	134	137	123	140	144	120	127
R	30	44	60	34	38	35	45	62	39	50	35	41

(ii) The following data gives the number of defectives in 10 independent samples of varying size from a production process. Construct a control chart for fraction defectives (p-chart),comment on state of control. (6)

sample No :	1	2	3	4	5	6	7	8	9	10
Sample size	90	65	85	70	80	80	70	95	90	75
Number of Defectives	9	7	3	2	9	5	3	9	6	7

12. a(i)Find the moment generating function of a Poisson distribution. Hence find mean and variance. (8)

(ii) If the probability density function of X is given by $f(x) = \begin{cases} 2(1-x), & for 0 < x < 1 \\ 0, & elsewhere \end{cases}$

Find $E[X^r]$ and $E[(2X+1)^2]$. (8)

(OR)

b(i) Find mean, variance and moment generating function of Exponential distribution. Also prove the memory lack property of Exponential distribution. (10)

(ii) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7. (1) What is the probability that the target would be hit on 10th attempt?(2) What is the probability that it takes him less than 4 shots? (6)

13.a(i)The Joint density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0<x<2, \quad 2<y<4 \\ 0, & otherwise \end{cases}$$

Find the marginal density functions of X and Y , also find $P(X < 1 / Y < 3)$ (8)

(ii)Find the coefficient of correlation between X and Y for the following probability distribution. (8)

X \ Y	0	1	2
0	0.1	0.2	0.1
1	0.2	0.3	0.1

(OR)

b(i) X and Y are two random variables having the joint probability mass function

$f(x,y) = \frac{1}{27}(2x+y)$, where x and y can assume only the integer values 0,1 and 2.
Find the marginal distributions of X and Y, conditional distribution of Y for X = x. (8)

(ii) The joint density function of two random variables is given $f(x,y) = e^{-(x+y)}$, $x > 0$, $y > 0$
Find the distribution of $\frac{X+Y}{2}$. (8)

14.a(i)The following are the number of sales which a sample of nine salespeople of Industrial Chemicals in California and a sample of six salespeople of Industrial Chemicals in Oregon made over a certain period of time:

California : 59, 68, 44, 71, 63, 46, 69, 54, 48

Oregon: 50, 36, 62 52, 70, 41

Assuming that the populations sampled can be approximated closely with normal distribution having the same variance, test the null hypothesis $\mu_1 = \mu_2$ against the alternative hypothesis $\mu_1 \neq \mu_2$ at 0.01 level of significance. (8)

(ii) Test for goodness of fit of a Poisson distribution at the 0.05 level of significance to the following data

X :	0	1	2	3	4	5
O_i :	275	138	75	7	4	1

(8)

(OR)

b(i) The following are the scores which random samples of students from two minority groups obtained on a current events test :

Minority Group 1: 73, 82, 39, 68, 91, 75, 89, 67, 50, 86, 57, 65, 70.

Minority Group 2: 51, 42, 36, 53, 88, 59, 49, 66, 25, 64, 18, 76, 74.

Use the U test (Wilcoxon test) at the 0.05 level of significance to test whether or not Students from the two minority groups can be expected to score equally well on the test. (8)

(ii) To determine whether 'efficiency' in job depends on the 'academic performance' 400 persons were examined yielding the following data:

Academic performance

efficiency		Excellent	Good	satisfactory
	Excellent	23	60	29
	Good	28	79	60
	satisfactory	9	49	63

Use the 0.01 level of significance to test whether null hypothesis that academic performance, is independent of efficiency. (8)

15(a) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following "cleanness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines:

	Engine1	Engine2	Engine3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Using ANOVA, test at the 0.01 level of significance whether there are differences in detergents or in the engines. (16)

(OR)

(b) The following are the numbers of mistakes made in five successive days by four technicians working for a photographic laboratory.

Technician I:	6,	14,	10,	8,	11.
Technician II:	14,	9,	12,	10,	14.
Technician III:	10,	12,	7,	15,	11.
Technician IV:	9,	12,	8,	10,	11.

Test at the 0.05 level of significance whether the differences among the four sample means can be attributed to chance. (16)

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