Reg. No. | | | | | | | | | |

B.E. / B.TECH. (FULL TIME) DEGREE END SEMESTER EXAMINATIONS – APRIL / MAY 2011

Common to B.E. / B. Tech. Mechanical, Printing, Agriculture & Irrigation, Chemical, PRPC, Auto & RPT, PT and Automobile

MA 9262 – Numerical Methods

20

(Regulations 2008)

FOURTH SEMESTER

Instructions: Only non-programmable calculators allowed

Time : 3 Hours **Answer ALL Questions** **Max. Marks : 100**

Part - A **(10 × 2 = 20)**

1. Find a positive roof of $x-2\sin x=0$ by Newton – Raphson method accurate upto 3 decimal places. Start with initial approximation $x_0=\frac{\pi}{2}$.

2. Find the dominant eigen value of $A=\begin{pmatrix}2 & 3\\ 5 & 4\end{pmatrix}$ by power method upto 1 decimal place accuracy. Start will $x^{(0)}=\begin{pmatrix}1\\ 1\end{pmatrix}$.

3. Find the Lagrange's interpolating polynomial passing through the points $(0,0),(1,1),(2,20)$.

4. Using Newton's forward difference interpolation formula find $y(1.5)$ given the following table:

x :	-1	0	1	2
y(x):	4	2	2	4

5. Using two-point Gaussian quadrature formula evaluate $\int_{-1}^{1}\frac{dx}{1+x^2}$ upto 4 decimal place accuracy.

6. Find the first derivative of y at $x=1.2$ using the following table:

x :	1.0	1.2	1.4	1.6	1.8
y:	2.7183	3.3201	4.0552	4.9530	6.0496

7. Using Taylor series find $y(0.1)$ correct to 4 decimal places. Take the first 5 terms of the series and $h=0.1$, given the differential equation $y'=x^2-y$, $y(0)=1$.

8. Using Euler's method compute $y(0.2)$ and $y(0.4)$ taking $h=0.2$, for the equation $y'=x+y$ with $y(0)=1$.

9. Write the finite difference scheme for $u_{xx}+u_{yy}=8x^2y^2$ for a square region with mesh size $\Delta x=\Delta y=1$.

10. Give the Bender – Schmidt scheme for $u_t=a^2u_{xx}$

Part - B **(5 × 16 = 80)**

11. (i) Using Gauss – Jordan method find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 4 & 1 & 0 \\ 2 & -1 & 3 \end{pmatrix}.$$ **(8)**

(ii) Find all the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$ by Jacobi's method. **(8)**

12. (a) (i) Using Newton's backward interpolation formula find the population for the year 1936 given that

year	x :	1901	1911	1921	1931	1941	1951
Population in thousands	y :	12	15	20	27	39	52

(8)

(ii) From the following table find $f(6)$ using Newton's divided difference formula

x :	1	2	7	8
$f(x)$:	1	5	5	4

(8)

(OR)

(b) The following values of x and y are given:

x :	1	2	3	4
y :	1	2	5	11

Find the cubic spline in the interval $1 \le x \le 2$ and hence evaluate $y(1.5)$ and $y'(1.5)$. **(16)**

13. (a) (i) The function $f(x, y)$ is defined by the following table:

y \ x	0	0.5	1.0	1.5	2.0
1	2.0	1.5	1.3	1.4	1.6
2	3.1	2.5	2.0	2.3	2.9
3	4.2	4.0	3.8	4.1	4.4

Compute $\int_1^3 \int_0^2 f(x, y)\, dx\, dy$ using Sompson's $\frac{1}{3}$ rule in both directions. **(8)**

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$ rule, taking $h = 1$. **(8)**

(OR)

(b) (i) The following table gives the values of $y = \frac{1}{1+x^2}$

x :	0	.125	.25	.375	.5	.675	.75	.875	1
y :	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5664	0.5

Take $h = .5, 0.25, 0.125$ and, use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2} dx$. Hence deduce an approximate value of π. **(10)**

(ii) Using backward difference find $y'(2.2)$ and $y''(2.2)$ form the following table.

x :	1.4	1.6	1.8	2.0	2.2
y :	4.0552	4.9530	6.0496	7.3891	9.0250

(6)

14. (a) Given $y'' = -xy' - y$, $y(0) = 1$, $y'(0) = 0$, find the value of $y(0.1)$ using Runge-Kutta method of order 4. Take $h = 0.1$. **(16)**

(OR)

(b) (i) Solve the equation $\frac{dy}{dx} = 1 - y$, given $y(0) = 0$, using Modified Euler's method and tabulate the solutions at $x = 0.1$, and 0.2, correct to 4 decimal places. Take $h = 0.1$. **(6)**

(ii) Given $\frac{dy}{dx} = \frac{1}{2}(1 + x^2) y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21$, evaluate $y(0.4)$ by Milne's predictor corrector method correct to 4 decimal places. **(10)**

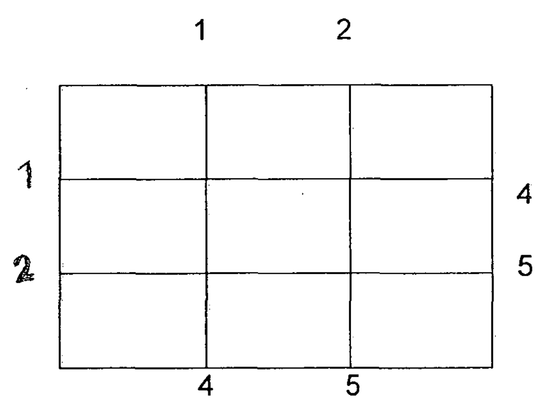
15. (a) (i) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0,t) = 0$, $u(4,t) = 0$ and the initial conditions $u(x,0) = x(4 - x)$ and $u_t(x,0) = 0$. Take $\Delta x = 1$ and $\Delta t = \frac{1}{2}$ and compute u for 2 time steps. **(8)**

(ii) Using Crank – Nicholson scheme solve $u_{xx} = 16u_t$, $0 < x < 1$, $t > 0$ given $u(x,0) = 0, u(0,t) = 0$, and $u(1,t) = 100t$. Take $\Delta x = \left(\frac{1}{4}\right)$ and $\Delta t = 1$. Compute u for one time step at the interior mesh points. **(8)**

(OR)

(b) (i) Solve $x^2 y'' + xy' + (x^2 - 3) y = 0$, $y(1) = 0$ and $y(2) = 2$ with $h = 0.25$ using finite different scheme. **(10)**

(ii) Solve $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in the figure below. Find the answer correct to 4 decimal points



(6)

&&&&&&&