21

Reg. No. | | | | | | | | | | |

B.E. / B. Tech DEGREE END SEMESTER EXAMINATION, APRIL/MAY 2011

Fourth Semester

MA 9265 – DISCRETE MATHEMATICS

Common to CSE & IT

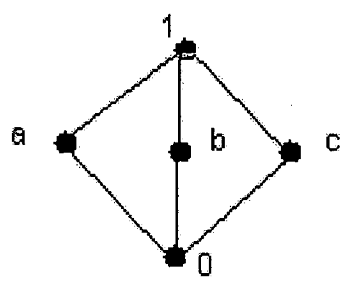
(Regulation 2008)

Time: Three hours **Maximum: 100 marks**

Answer ALL questions

Part – A ($10 \times 2 = 20$ marks)

1. Consider the statement "Given any integer, there is a greater positive integer". Symbolize this statement with and without using the set of positive integers as the universe of discourse.
2. Prove that if n is an integer and $n^3 + 5$ is odd, then n is even using a proof by contradiction.
3. Show that among any group of five (not necessarily consecutive) integers there are two integers with the same reminder when divided by 4.
4. How many bit strings of length ten contain (i) exactly four 1's, (ii) at least four 1's?
5. If G is self complementary graph, then prove that G has $n \equiv 0\ (or)\ 1 \pmod 4$ vertices.
6. If G is multi connected graph has an Euler Circuit, then prove that every vertex of G is even.
7. Is every Abelian group is cyclic? Justify your answer.
8. Prove that the order of every element of a finite group is finite.
9. Prove that the greatest lower bound of any two elements is unique in a poset, if it exists.
10. Find whether the cancellation law is valid in the following poset.



Part – B ($5 \times 16 = 80$ marks)

11. i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2,3, 5 and 6. (8)

ii) Solve the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$. (8)

12. (a) i) Use rules of inferences to obtain the conclusion of the following arguments:
"Sachin, a student in this class, knows how to write programmes in JAVA".
"Everyone who knows how to write programmes in JAVA can get a high-paying job".
Therefore, "Someone in this class can get a high-paying job". (6)

ii) Show that $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$ by using equivalences. (5)

iii) Prove that "$\sqrt{2}$ is irrational" by giving a proof by contradiction. (5)

(OR)

12. (b) i) Show that $(\forall x)(P(x) \to Q(x)) \wedge (\forall x)(Q(x) \to R(x)) \Rightarrow (\forall x)(P(x) \to R(x))$. (6)

ii) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology. (5)

iii) Is the following argument valid? Justify.
"If Babu does every problem in the discrete mathematics book, then he will learn discrete mathematics". "Babu learned discrete mathematics".
Therefore, "Babu did every problem in the discrete mathematics book". (5)

13. a) i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (6)

ii) Are the simple graphs with the following adjacency matrices isomorphic?

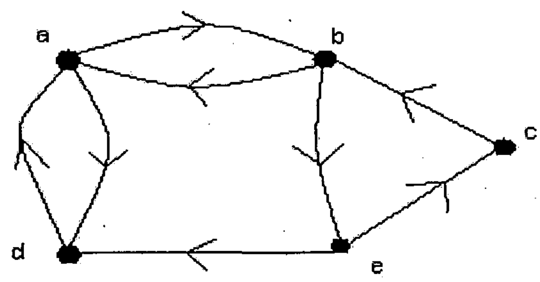
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(5)

iii) Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. (5)

(OR)

13. (b) i) Find the number of walks (elementary paths) of length 4 from the vertex a to the vertex e in the following directed graph using the adjacency matrix. (6)



ii) Prove that the number of odd degree vertices in any graph is even. (5)

iii) Show that a simple graph with n vertices is connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (5)

14. (a) i) Let G be a finite cyclic group generated by an element $a \in G$. If G is of order n, then prove that $a^n = e$, so that $G = \{a, a^2, a^3, ..., a^n = e\}$. Further, n is the least positive integer for which $a^n = e$. (6)

ii) Prove that the order of a subgroup of a finite group divides the order of the group. (10)

(OR)

14. (b) i) Prove that every finite group of order n is isomorphic to a permutation group of order n. (8)

ii) Let $f: G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$.
Prove that (1) the kernel of f is normal subgroup of G
(2) $f(a^{-1}) = [f(a)]^{-1}$, for every $a \in G$. (8)

15. (a) i) Prove that every chain is a lattice. Further, prove that it is a distributive lattice. Find whether every distributive lattice is a chain. Justify your answer. (8)

ii) Show that following in a complemented distributive lattice.
$a \le b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \vee b = 1 \Leftrightarrow b' \le a'$.
Where a' denotes the complement of a. (8)

(OR)

15. (b) i) Let $\langle L, \le \rangle$ ordered lattice and $a \wedge b = \text{glb}(a,b)$ & $a \vee b = \text{lub}(a,b)$.
Prove that $\wedge$ and $\vee$ satisfies the following properties
(1) Idempotent, (2) Associative, (3) Commutative, (4) Absorption. (8)

ii) Show that in a Boolean algebra
1) The De Morgan's laws are valid
2) $a \cdot b' + b \cdot c' + c \cdot a' = a' \cdot b + b' \cdot c + c' \cdot a$ (8)

*****