Roll No.

B.E/B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY 2011

SIXTH SEMESTER **(REGULATION 2008)**

MA9266 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science Engineering and Computer Technology)

Time : 3 hrs Max. Marks : 100

Answer ALL Questions

Part – A (10 x 2 = 20 Marks)

1. Let $P(X=x)=\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x=1,2,3,\ldots$, be the probability mass function of a random variable X. Find $P(X \geq 4)$.

2. The random variable X has the p.d.f. $f(x)=\frac{1}{3}e^{-\frac{1}{3}x}, x>0$, Find the moment generating function $M_X(t)$ of X and hence obtain $E(X)$.

3. If the joint p.d.f. of the random variable X and Y is given by

$$f(x,y)=\begin{cases} 10xy^2, & 0<x<y<1 \\ 0 & otherwise \end{cases}$$, find the marginal p.d.f. of the random variable X.

4. State the central limit theorem for independent and identically distributed random variables.

5. Derive the autocorrelation function for a Poisson process with rate λ.

6. Let $\{X_n : n \geq 0\}$ be a Markov chain having state space $S=\{0,1\}$ and one-step transition probability matrix $P=\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.

7. Consider an $M/M/1/\infty$ queueing system. If $\lambda=6$ and $\mu=8$, find (i) the probability of no customers in the system and (ii) the probability of atleast 5 customers in the system.

8. Compute P_0 and P_N for $\lambda = 2$ /minutes , $\mu = 45$ / hour, $C = 2$ and $N = 6$ using $M/M/C/N/FCFS$ queueing system.

9. In an $M/D/1$ queueing system, an arrival rate of customer is 10 per second and a service rate of customers is 20 per second. Compute the mean number of customers in the system and the probability that the service is idle.

10. For an 4 queue tandem Markovian networks with service rates $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu$ and arrival rate λ, compute the mean number of customers in the system and delay a customer experiences in passing through networks.

PART – B (5 x 16 = 80 Marks)

11. (i) If a random variable X has the p.d.f. $f(x) = \begin{cases} x e^{-x}, & x > 0 \\ 0 & x \le 0 \end{cases}$,

find (1) Moment generating function $M_X(t)$ of X and hence obtain $E(X)$ and $Var(X)$ (2) Cumulative distribution function of X (3) $P(X > 2)$.

(ii) The random variable X has geometric distribution $P(X = n) = p(1-p)^{n-1}$, $n = 1, 2, 3, \ldots$. Show that $P(X > m + n / X > n) = P(X > m)$.

(iii) Let X be a uniformly distributed random variable over $\left(-\pi/2, \pi/2\right)$. Find the p.d.f. of the random variable Y if $Y = \sin X$.

12. a.(i) The random variables X and Y have the following joint p.d.f.

$$f(x, y) = \begin{cases} 2e^{-x}e^{-y}, & 0 < y < x < \infty \\ 0 \quad , & elsewhere \end{cases}.$$

Compute the correlation coefficient ρ_{XY} of X and Y.

(ii) If X and Y are two random variables with joint p.d.f. given by $f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0 \quad , & otherwise \end{cases}$, find the p.d.f. of the random variable $U = \dfrac{X}{Y}$.

(OR)

b.(i) If the joint p.d.f. of the random variable (X, Y) is $f(x, y) = \begin{cases} 8xy, & 0 < y < x, \ 0 < x < 1 \\ 0 \ , & otherwise \end{cases}$,

find (1) the marginal p.d.f. of X (2) the conditional p.d.f. of Y given $X = x$

(3) $P\left(Y < 1/8 \ / \ X < 1/2\right)$.

(ii) Two random variables X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} \dfrac{3}{2}\left(x^2 + y^2\right), & 0 \le x \le 1, 0 \le y \le 1 \\ 0 \quad , & otherwise \end{cases}.$$

Find the correlation coefficient ρ_{XY} of X and Y.

13. a.(i) A random process is given by $X(t) = U + V\cos(\omega t + \phi)$ where U is a random variable that is uniformly distributed between -2 and 2, V is a random variable with $E(V) = 0$ and $Var(V) = 2$, ω is a constant and ϕ is a random variable that is uniformly distributed from $-\pi$ to π. Here U, V and ϕ are independent random variables. Compute $E(X(t))$, $E(X(t)X(t+\tau))$ and $Var(X(t))$. Is the process $X(t)$ stationary in wide sense? Explain.

(ii) Let $\{X_n : n = 0,1,2,\ldots\}$ be a Markov chain having state space $S = \{1,2,3\}$ with one-step transition probability matrix $P = \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{bmatrix}$ and the initial distribution $P(X_0 = i) = 1/3$, $i = 1,2,3$. Find (1) $P\{X_3 = 1, X_2 = 1, X_1 = 1, X_0 = 2\}$

(2) $P(X_2 = 2, X_1 = 1/X_0 = 1)$ (3) $P(X_3 = 3/X_2 = 2, X_1 = 1, X_0 = 3)$

(4) $P(X_2 = 3, X_0 = 3)$.

(OR)

b.(i) Show that the interarrival time between two consecutive arrivals in a Poisson process is an exponential random variable. Is the Poisson process stationary? Explain.

(ii) Consider a Markov chain $\{X_n : n \geq 0\}$ having state space $S = \{1,2,3,4\}$ and one-step transition probability matrix

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}.$$

(1) Draw a transition diagram.
(2) Is the chain irreducible? Explain.
(3) Is the state – 2 ergodic? Explain.
(4) Is the state – 3 transient?

14. a.(i) For an $M/M/C/\infty$ queue, derive the system of differential difference equations for the system size probabilities. Calculate the steady – state probabilities of the number of customers in the system under steady state condition.

(ii) A one person barber shop has 6 chairs to accommodate people waiting for a hair cut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the rate of 3 per hour and spend an average of 15 minutes in the barber's chair. Find (1) P_0 (2) L_q (3) P_7 (4) W_S.

(OR)

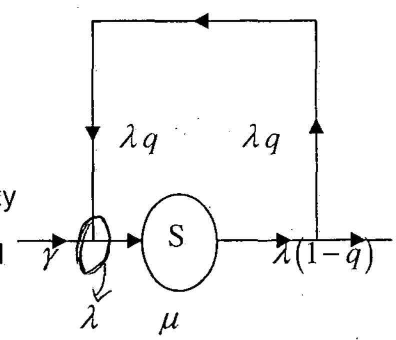
b.(i) For an $M/M/1/N$ queue, derive the system of differential difference equation for the system size probabilities. Calculate the steady-state probabilities of the number of customers in the system and mean number of customers in the system.

(ii) In an $M/M/1/\infty$ queueing system, if $\lambda = 10$ and $\mu = 15$, compute

(1) L_q (2) W_S (3) P_3 (4) Probability that an arriving customer has to wait in the queue. (5) On the average, how many customers will be served by the server.

15. a.(i) Discuss an $M/G/1/\infty$ queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula.

(ii) The simplest open network consists of a single node with feedback as illustrated in the figure. Let μ be the service rate per job. The external arrival process has rate γ. After completing service, jobs return to the queue with probability 'q' and leave the system with probability '1-q'. Find the total arrival rate λ and stability condition for the system.



(OR)

b. Discuss the open queueing network system and hence obtain
(1) the product form solution of the system size probabilities
(2) mean number of customers in the system.
