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B.E / B.Tech (Full Time) Degree End Semester Examinations APRIL/MAY 2013

Common to All Branches

Second Semester

MA 8251- Mathematics II

(Regulations 2012)

Duration: 3 Hours — Total marks:100

Answer ALL Questions

Part A (10 x 2 = 20 Marks)

1. Find the particular integral of $(D^2+6D+9)y = e^{-2x}x^3$.

2. Guess the trial solution of the particular integral for the differential equation $y'' - y = e^x \sin 2x$ using method of undetermined coefficients.

3. Find the directional derivative of $\phi = 2yz + z^2$ at the point (1,-1,3) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

4. If $\vec{F} = (4xy - 3x^2z^2)\vec{i} + 2x^2\vec{j} - 2x^3z\,\vec{k}$, check whether the integral $\oint_C \vec{F}\cdot d\vec{r}$ is independent or dependent of the path C.

5. Show that an analytic function with constant imaginary part is constant.

6. Find the image of $|z| = 1$ under the mapping $w = z + (2+2i)$.

7. Evaluate $\int_C \frac{dz}{(z+1)^2(z-2)}$, where C is the circle $|z| = 3/2$.

8. Identify and classify the singularity of the function $f(z) = (\sin z - z)/z^3$.

9. Find the Laplace transforms of $f(t) = te^{-t}\sin t$.

10. Using intial value theorem, find $\lim_{t\to 0} f(t)$ given that $L\{f(t)\} = F(s) = \frac{1}{(s+1)^2+1}$.

Part B (5 X 16 = 80 Marks)

11.. (i) Using method of variation of parameters solve the following differential equation $y''+4y=4\tan 2x$. (8 Marks)

11. (ii) Solve $(x^2D^2-3xD+5)y=x^2\sin(\log x)$. (8 Marks)

12.(a). (i). Verify Green's theorem for $\oint_C [(x^2-xy^3)dx+(y^2-2xy)dy$ where C is the boundary of the square in the xy-plane (0,0), (2,0), (2,2), (0,2). (8 Marks)

(ii). Verify Stokes' theorem for the vector field $\vec{F}$ = $(2x-y)\vec{\imath}-yz^2\vec{\jmath}-y^2z\vec{k}$ over the upper half surface S : $x^2+y^2+z^2=1$ bounded by its projection on the xy-plane. (8 Marks)

(OR)

(b). (i). Verify Gauss divergence theorem for the vector function $\vec{F}=x^2\vec{\imath}+z\vec{\jmath}+yz\vec{k}$ over the cube bounded by x=1, x= -1, y=1, y= -1, z=1 and z= -1. (10 Marks)

(ii). Verify that $\vec{F}=(y^2\cos x+z^3)\vec{\imath}+(2y\,sinx-4)\vec{\jmath}+3xz^2\vec{k}$ is irrotational; further find also its corresponding scalar potential. (6 Marks)

13. (a). (i). Find the analytic function f(z) = u(x; y)+iv(x; y), given that

v(x, y) = $e^{2x}(x\cos 2y-y\sin 2y)$. (8 Marks)

(ii). Find the bilinear transformation which maps the points z = ∞, i, 0 into the points w = 0, -i, ∞ respectively. (8 Marks)

(OR)

(b). (i). Under the mapping w = 2z find the image (in w-plane) of the triangular region bounded by the lines x = 0; y = 0 and x+y = 1 in z-plane. (8 Marks)

(ii). Find the image of the circle $|z-3i|=3$ under the map w = 1/z. (8 Marks)

14. (a). (i). Find the Laurent's expansion of $f(z)=\frac{z^2-1}{(z+3)(z+2)}$ valid in the region

(i). $2<|z|<3$ and (ii). $|z|>3$. (8 Marks)

(ii). Using contour integration on unit circle evaluate, $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$. (8 Marks)

(OR)

(b). (i). Using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. (8 Marks)

(ii). Using Cauchy's residue theorem, evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C: $|z| = 4$.

(8 Marks)

15. (a). (i). Solve $y''-4y'+3y = e^{-t}$; $y(0) = 1$, $y'(0) = 0$ by Laplace transforms method.

(8 Marks)

(ii). Using convolution theorem, find the inverse Laplace transforms of $\left[\frac{1}{(s+1)(s^2+1)}\right]$. (8 Marks)

(OR)

(b). (i). Find the Laplace transforms of the periodic function defined on the interval $0 \leq t \leq \frac{2\pi}{\omega}$ by $f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}. \end{cases}$ and $f(t+\frac{2\pi}{\omega}) = f(t)$.

(8 Marks)

(ii). Find the inverse Laplace transforms of $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$. (8 Marks)

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