B.E / B.Tech (Part Time) DEGREE END SEMESTER EXAMINATIONS, OCT / NOV 2013

Computer Science and Engineering

Semester - IV

PTMA 040 / PTMA 505 PROBABILITY AND QUEUEING THEORY

(Regulation 2005)
(Statistical Tables are permitted into the Examination Hall)

Time : 3 Hours | Answer ALL Questions | Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. In a certain region of Russia, the probability that a person lives at least 80 years is 0.75, and the probability that he or she lives at least 90 years is 0.63. What is the probability that a randomly selected 80 year old person from this region will survive to become 90?
2. From an ordinary deck of 52 cards, cards are drawn one by one, at random and without replacement. What is the probability that the fourth heart is drawn on the tenth draw?
3. A certain basket ball player makes a foul shot with probability 0.45. Determine for what values of k the probability of k baskets in 10 shots is maximum.
4. Let X be a gamma random variable with parameters (r,λ). Find the distribution function of cX, where c is a positive constant.
5. First a point Y is selected at random from an interval $(0,1)$. Then another point X is selected at random from the interval $(Y,1)$. Find the probability density function of X.
6. Show that $X-Y$ and $X+Y$ are uncorrelated if and only if $Var(X)=Var(Y)$.
7. A particle performs a random walk with absorbing barriers at 0 and 4. Whenever it is at any position $r\,(0<r<4)$ it moves to $r+1$ with probability p or $r-1$ with probability q such that $p+q=1$. Find the transition probability matrix.
8. Suppose that eqrthquakes occur in a certain region of California in accordance with a Poisson process at a rate of 7 per year.
   1. What is the probability of no earthquakes in 1 year?
   2. What is the probability that in exactly 3 of the next eight years no earthquakes will occur?
9. State and Explain the Kendall's notation for a Generalized Poisson Queue model.
10. For a $M/G/1$ model, obtain the Pollazeck-Khinchine formula when the service times are deterministic.

Part – B (5 x 16 = 80 marks)

11. (i) A Judge is 65% sure that a suspect has committed a crime. During the course of the trial a witness convinces the judge that there is an 85% chance that the criminal is left-handed. If 23% of the population is left-handed and the suspect is also left-handed with this new information, how certain should the judge be of the guilt of the suspect? **(6)**

    (ii) The distribution function of a random variable X is given by

$$F(x)=\begin{cases}0 & x<0\\ \frac{x}{4} & 0\le x<1\\ \frac{1}{2} & 1\le x<2\\ \frac{x}{12}+\frac{1}{2} & 2\le x<3\\ 1 & x\ge 3\end{cases}$$

Compute:

(1) $P\{X<2\}$, (2) $P\{X=2\}$, (3) $P\{1\le X<3\}$, (4) $P\left\{X>\frac{3}{2}\right\}$, (5) $P\left\{X=\frac{5}{2}\right\}$,
(6) $P\{2<X\le 7\}$ **(6)**

(iii) The sales of a convenience store on a randomly selected day are X thousand dollars, where X is a random variable with a distribution function of the following form:

$$F(t)=\begin{cases}0 & t<0\\ \frac{t^2}{2} & 0\le t<1\\ k\,(4t-t^2) & 1\le t<2\\ 1 & t\ge 2\end{cases}$$

Suppose that this convenience store's total sales on any given day are less than \$2000. (1) Find the value of k. (2) Let A and B be the events that tomorrow the store's total sales are between 500 and 1500 dollars and over 1000 dollars, respectively. Find $P(A)$ and $P(B)$. (3) Are A and B independent events? **(4)**

12. a) (i) Of police academy applicants, only 25% will pass all the examinations. Suppose that 12 successful candidates are needed. What is the probability that, by examining 20 candidates, the academy finds all of the 12 persons needed? **(6)**
(ii) The scores on an achievement test given to 1, 00,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students? **(6)**
(iii) Show that an exponential distribution is memory less. **(4)**

OR

b) (i) A father asks his sons to cut their backyard lawn. Since he does not specify which of the 3 sons is to do the job, each boy tosses a coin to determine the odd person, who must then cut the lawn. In the case that all three get heads or tails, they continue tossing until they reach a decision. Let p be the probability of heads and $q=1-p$, the probability of tails.
(1) Find the probability that they reach a decision in less than n tosses. **(4)**
(2) If $p=\frac{1}{2}$, what is the minimum number of tosses required to reach a decision with probability 0.95? **(4)**
(ii) Determine the moment generating function of a Gamma Random variable X with parameters (n,λ). Hence obtain the mean and variance of X. **(8)**

13. a) Show that if X and Y are continuous random variables with joint probability density function $f(x,y)=\frac{1}{3}(x+y), 0\le x\le 1, 0\le y\le 2$. Obtain the two lines of regression. **(16)**

OR

b) (i) Customers arrive at a mall in accordance with a Poisson process with rate 4000 persons per day. Find an approximate value for the probability that tomorrow at least 3850 customers will enter the mall. **(6)**

(ii) Let X and Y be jointly distributed with joint probability density function

$$f(x,y)=\frac{1}{2}x^3e^{-xy-x},\ x\geq 0, y\geq 0.$$

Determine if X and Y are positively correlated, negatively correlated or uncorrelated. **(10)**

14. a) State the postulates for a Poisson process: $\{N(t), t\geq 0\}$ Using the postulates obtain the probability distribution

$$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!},\ n=0,1,2\ldots;\ \lambda>0.$$ **(16)**

OR

b) Given the transition probability matrix $P=\begin{pmatrix}0 & \frac{2}{3} & \frac{1}{3}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$ for a Markov chain with state space $S=\{1,2,3\}$. Classify the states and obtain the stationary distribution if it exists. **(16)**

15. a) (i) Derive the steady state probabilities for $M|M|1:FCFS|N|\infty$ model and hence find L_s **(10)**

(ii) Two repairmen are attending five machines in a workshop. Each machine breaks down according to a Poisson distribution with mean 3 per hour. The repair time per machine is exponential with mean 15 minutes. Find the probability that the two repairmen are idle, that one repairman is idle and what is the expected number of idle machines not being serviced? **(6)**

OR

b) Obtain the Pollaczek–Khintchine(P-K) formula for the $M|G|1:GD|\infty|\infty$ model **(16)**