

Reg. No. :

B.E. (Full Time) END SEMESTER EXAMINATIONS APRIL/MAY 2011

Branch : Computer Science and Engineering

SEMESTER : SIXTH

REGULATIONS – 2008

CS 9032 – GRAPH THEORY

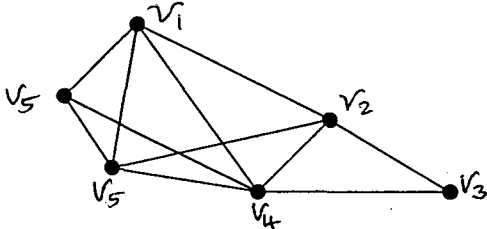
Time : 3 hours

Max. Marks : 100

Answer ALL Questions

PART A – (10 x 2 = 20 marks)

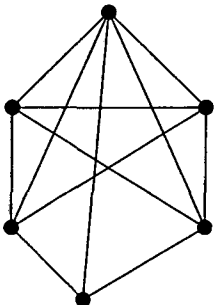
1. Draw a picture of the following graph, and state whether it is simple or not.  $G = (V, E)$ , where  $V = \{v_1, v_2, v_3, v_4, v_5\}$  and  $E = \{v_1v_1, v_1v_2, v_2v_3, v_3v_4, v_5v_4, v_4v_5\}$ .
2. Suppose you are given eight coins and are told that seven of them are of equal weight, and one coin is either heavier or lighter than the rest. You are provided with an equal-arm balance, which you may use only three times, for comparing coins. Sketch a strategy in the form of a decision tree for identifying the nonconforming coin, as well as for finding out whether it is heavier or lighter than the rest.
3. Is the following graph planar? If yes, show a plane representation.



4. What is 1-isomorphism?
5. Draw the graph whose incidence matrix is given below:

1	1	1	1	0	0	0	0
1	1	0	0	1	1	0	0
0	0	0	1	0	1	1	1
0	0	1	0	1	0	1	1

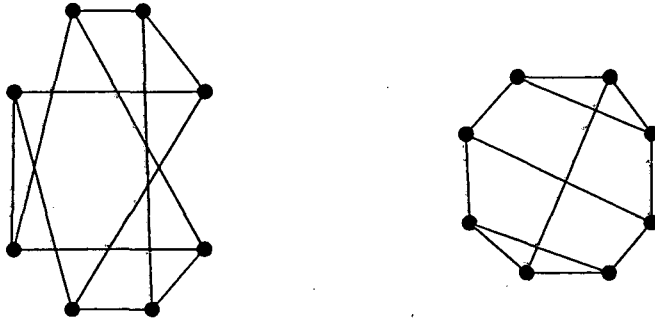
6. Find the chromatic number of the following graph:



7. When repeated fusion of adjacent vertices is done to find whether a graph is connected or not, what will be the maximum number of fusions that need to be done when there are  $n$  vertices in the graph?
8. Write in brief on any one method to find if a graph is separable or not.
9. What is meant by a spanning arborescence?
10. What is an invariant of a graph?

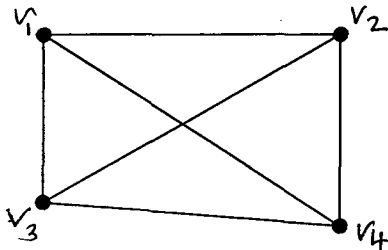
**PART B – (5 x 16 = 80 marks)**

11. a) If you flip a coin three times, there are 8 ways the coins can come up: HHH, HHT, HTH, etc.... Suppose these are the labels on 8 vertices of a graph, and that you draw an edge between a pair of vertices if the labels differ in exactly one place. For example, HTH will be adjacent to HTT, but will not be adjacent to THH. Find if the resulting graph is isomorphic to any or all of the following graphs. (8)



- b) Prove that a given connected graph  $G$  is an Euler graph if and only if all vertices of  $G$  are of even degree. (8)

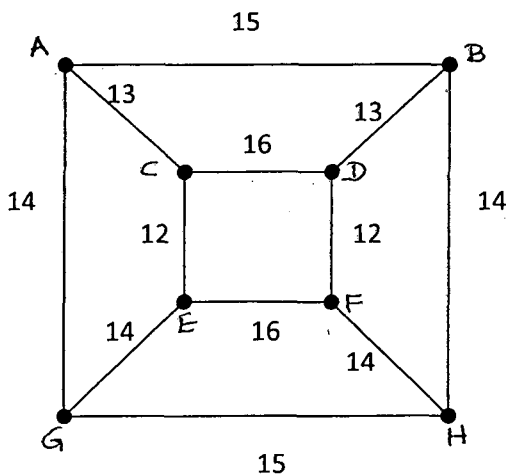
12. a) (i) Find all the spanning trees of the following graph using cyclic interchange. Find the distance between the first and the last spanning trees you have found. (8)



- (ii) Prove that every circuit has an even number of edges in common with any cut-set. (8)

(OR)

12. b) (i) Apply Prim's algorithm to find a minimal spanning tree for the graph given below. Explain the procedure step-by-step. (8)

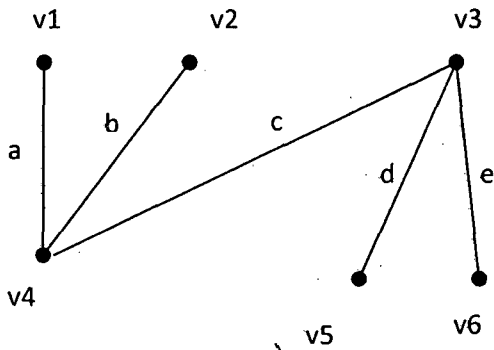


- (ii) Prove that with respect to a given spanning tree  $T$ , a branch  $b_i$  that determines a fundamental cut-set  $S$  is contained in every fundamental circuit associated with the chords in  $S$ , and in no others. (8)

13. a) (i) Check whether the following statement is true or false. Justify your answer by giving a short proof or counter example. "Every bipartite graph need not be a tree." (8)

(ii) In the graph given below give the following with justification: (8)

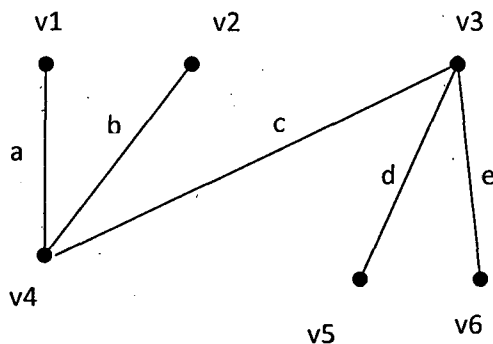
- I) A matching of maximum size
- II) A minimal dominating set
- III) An independent set of vertices of maximum size
- IV) A minimal edge covering



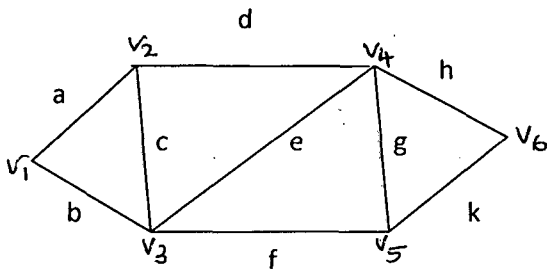
(OR)

13. b) (i) Sketch any one orientation of a complete graph of four vertices. Characterize the digraph in terms of binary relations. (8)

(ii) Find all the minimal dominating sets for the following graph using the method that uses Boolean arithmetic: (8)

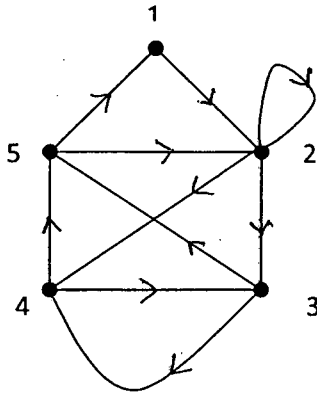


14. a) Use Paton's algorithm to find all the fundamental circuits in the following graph. Show the step-by-step procedure. (16)

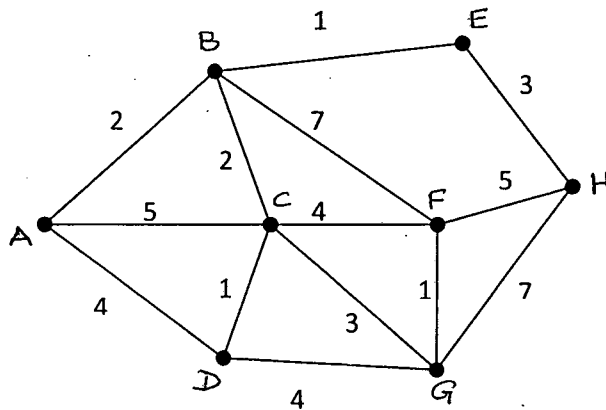


(OR)

14. b) Use Roberts and Flores' algorithm to find all the directed circuits in the following graph. Show the step-by-step procedure. (16)



15. a) Use Dijkstra's algorithm to find the shortest distance between the vertices A and H in the graph given below. Explain the step-by-step procedure. The distances between adjacent vertices is shown in the graph. (16)



(OR)

15. b) Explain the planarity testing algorithm proposed by Hopcroft and Tarjan. Discuss on the technique they have used to resolve ambiguity in adding paths. (16)