REG. NO.

B.E. / B.TECH (FULL TIME) DEGREE END SEMESTER EXAMINATION,

MARCH 2011

CIVIL ENGINEERING

FIRST SEMESTER

MA 171 MATHEMATICS - I

(COMMON TO ALL BRANCHES)

(REGULATION 2004)

TIME: THREE HOURS **MAX.:100 MARKS**

Answer ALL Questions

PART-A (10 x 2 = 20 Marks)

1. If λ is an eigenvalue of a non-singular matrix A, show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.
2. If the sum of two eigenvalues and trace of a square matrix A of order 3 are equal, find the value of $|A|$.
3. Find the equation to the sphere, having the points(-4, 5, 1) and (4, 1,7) as ends of a diameter.
4. Define a tangent line.
5. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$.
6. State the convergence of binomial series.
7. If $u = x^3 y^4$, where $x = t^3$ and $y = t^2$, find $\frac{dy}{dt}$ in terms of t.
8. Write the Taylor's series expansion of a function $f(x,y)$ in powers of $x-a$ and $y-b$.
9. Solve $(D^2 + 2D + 3)y = 0$
10. Find the particular integral of $(D^2 + 4D)y = \sin 2x$.

PART-B (5 x 16 = 80 Marks)

11. (i)Use Cayley-Hamilton theorem to find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$, where $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$. (8)

(ii) Reduce quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 12yz + 8zx$ into canonical form by orthogonal transformation. (8)

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12(a)(i) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$; $x - 2y + 2z = 5$ for a great circle. (8)

(ii) Find the equation of the cone whose vertex is origin and guiding curve the circle $x^2 + y^2 + z^2 + 2x - y + 3z - 1 = 0$, $x - y + z + 4 = 0$. (8)

(OR)

(b)(i) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z - 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. (8)

(ii) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$. (8)

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13(a)(i) Test for convergence the series $1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \ldots\ldots$ By D'Alembert's Ratio test. (8)

(ii) Examine the character of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$. (8)

(OR)

(b)(i) Test for convergence the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots$ $(0 < x < 1)$ (8)

(ii) Find the value of x in which the series $\frac{1}{(1-x)} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \ldots$ converge. (8)

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14(a)(i) If z is a function of x and y, where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. (8)

(ii)Find the maxima and minima of $f(x,y) = x^3 + y^3 - 3xy$. (8)

(OR)

(b)(i) If $p = 3x + 2y - z, q = x - 2y + z, r = x + 2y - z$, find $\frac{\partial(p,q,r)}{\partial(x,y,z)}$. (6)

(ii) Find the dimension of a rectangular box without top of maximum capacity given the surface area as 432 square meters. (10)

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15(a)(i) Solve $(D^2 - 2D + 1)y = xe^x \sin x$. (8)

(ii) Solve $(D^2 - 2D + 1)y = e^x \log x$ by method or variation of parameters. (8)

(OR)

(b)(i) Solve $\frac{dx}{dt} + y = \sin t$, $x + \frac{dy}{dt} = \cos t$ given that $x = 2$ and $y = 0$ at $t = 0$. (8)

(ii) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (8)

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