Registration Number | | | | | | | | | |

B.E./B.Tech (Full Time)DEGREE EXAMINATION - APRIL/MAY 2013
FOURTH SEMESTER-(REGULATION 2008)

(Common to Mechanical, Electrical and Electronics, Agriculture and Irrigation, Printing, Chemical, PRPC, Pharmaceutical, Production, Automobile, Aeronautical and Rubber and Plastic)

MA 9262 NUMERICAL METHODS

Max Marks: 100 **ANSWER ALL QUESTIONS** **Time: 3 Hours**

PART A ($10 \times 2 = 20$ Marks)

1. What is the difference between direct and iterative methods of solving simultaneous linear equations?

2. Write down the Newton-Raphson algorithm for finding the reciprocal of N.

3. Is Newtons divided difference formula symmetric in its arguments? Justify.

4. Test whether the following functions are cubic splines or not?
   $p_1(x) = -2x^2 + x^3, \; -1 \leq x \leq 0$
   $p_2(x) = x^2 - 2x^3, \; 0 \leq x \leq 1.$

5. State the three-point Gaussian quadrature formula. For what class of functions $f(x)$ does it give exact answers?

6. Compare Trapezoidal rule and Simpsons one-third rule for evaluating numerical integration.

7. State the special advantage of Runge-Kutta method over taylor series method.

8. How to proceed, if there is a considerable difference between predicted value and corrected value, in predictor corrector methods?

9. Write the central difference approximation for $y'(x)$ and a finite difference approximation for $y''(x)$.

10. What will be the value of u at an interior grid point if u satisfies Laplace equation and $u = 100$ on the boundary of a square.

PART B (5 × 16 = 80 Marks)

11. (a) i. Find the first and second derivatives of $f(x)$ at $x = 1.5$ if

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.000	13.625	24.000	38.875	59.000

(8)

ii. Apply Trapezoidal rule to evaluate $\int_1^5 \int_1^5 \frac{dxdy}{\sqrt{(x^2+y^2)}}$, taking two sub-intervals. (8)

12. (a) i. Solve the equations $2x + 4y + z = 3$; $3x + 2y - 2z = -2$; $x - y + z = 6$ by Gauss ellimination method. (8)

ii. Use Gauss-Jordan method to find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$. (8)

(OR)

(b) i. Using Jacobi's method, find all the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$. (10)

ii. Solve the equations $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ by Jacobi's method correct to three decimal places. (6)

13. (a) i. Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the following values of x and y are given:

x	5	6	9	11
y	12	13	14	16

(8)

ii. If $u_{-1} = 10$, $u_1 = 8$, $u_2 = 10$, $u_4 = 50$, find u_0 and u_3. (8)

(OR)

(b) i. Find the cubic splines for the following table of values:

x	1	2	3
y	−6	−1	16

Hence evaluate $y(1.5)$ and $y'(2)$. (10)

ii. Certain corresponding values of x and $\log_{10} x$ are given below:

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

Find $\log_{10} 310$ by Newton's divided difference formula. (6)

14. (a) i. Solve the initial value problem $\frac{dy}{dx} = 1 + xy^2$, $y(0) = 1$ for $x = 0.4$ by using Milne's method, when it is given that

x	0.1	0.2	0.3
y	1.105	1.223	1.355

(10)

ii. Evaluate $y(0.1)$ correct to four decimal places using Taylor's series method if $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. (6)

(OR)

(b) i. Find $y(0.4)$ by Adams-Bashforth method given $\frac{dy}{dx} = \frac{1}{2}xy$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$. (8)

ii. Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$, with initial condition $y = 1$ at $x = 0$, for the range $0 \le x \le 0.6$ in steps of 0.2. (8)

15. (a) i. Solve the boundary value problem $y'' - 64y + 10 = 0$ with $y(0) = y(1) = 0$ by the finite difference method taking $h = \frac{1}{4}$. (8)

ii. Obtain the solution of the equation $\nabla^2 u = 8x^2y^2$ in the square region $-1 \le x,\ y \le 1$ with $u = 0$ on the boundaries, taking $\Delta x = \Delta y = 0.5$. (8)

(OR)

(b) i. Solve by Crank-Nicolson's scheme: $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$. $u(x, 0) = 100 \sin \pi x$, $u(0, t) = u(1, t) = 0$. Compute u to the nearest integer for one time step with $h = \frac{1}{4}$. (8)

ii. Find the solution of the hyperbolic equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ when $0 < x < 1$, $t > 0$, $u(x, 0) = 100(x - x^2)$, $\frac{\partial u}{\partial t}(x, 0) = 0$; $u(0, t) = u(1, t) = 0$. Taking $h = 0.25$, compute u for 4 time steps. (8)

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