Reg. No. [ ]

END SEMESTER EXAMINATIONS – MAY 2013
FOURTH SEMESTER
B.E. / B.TECH
MA9261 PROBABILITY AND STATISTICS

[Common to Geo-Informatics, Manufacturing Engineering, Industrial Engineering, Textile Technology, Industrial Bio-technology, Food Technology, Pharmaceutical Technology and Apparel Technology]

USE OF STATISTICAL TABLES IS PERMITTED

Time : 3 Hours | **Answer ALL Questions** | **Max. Marks : 100**

Part - A **(10 × 2 = 20)**

1. If the probability density function of the random variable X is given by $f(x)=\begin{cases} kx^3, & 0<x<1 \\ 0, & elsewhere \end{cases}$ find the value of k
2. Find the mean of Geometric distribution.
3. Define correlation coefficient.
4. A random sample of size $n=100$ is taken from an infinite population with the mean $\mu=75$ and the variance $\sigma^2=256$. Based on central limit theorem what is the probability that the mean of the sample $\overline{X}$ will fall between 67 and 83?
5. Suppose that 100 tires made by a certain manufacturer lasted on the average 21,819 miles with a standard deviation of 1,295 miles. Test the null hypothesis $\mu=22{,}000$ miles against the alternative hypothesis $\mu<22{,}000$ miles at the 0.05 level of significance.
6. Nine determinations of specific heat of iron had a standard deviation of 0.0086. Test the null hypothesis that $\sigma=0.01$ for such determinations. Use the alternative hypothesis $\sigma\neq 0.01$ at the 0.05 level of significance.
7. State the identity for sum of squares for two- way of analysis of variance.
8. Write any one difference between Randomized block design and Latin square Design.
9. A garment was sampled on 10 consecutive hours of production. The number of defects found per garment is given below: 5,1,7,0,2,3,4,0,3,2. Compute upper and lower control limits for monitoring number of defects.
10. Define tolerance limits.

Part - B **(5 × 16 = 80)**

11. (i) The Joint density function of two random variables X and Y is given by

$$f(x,y)=\begin{cases} xy^2+\dfrac{x^2}{8}, & 0<x<2, \quad 0<y<1 \\ 0, & otherwise \end{cases}$$

Find the marginal density functions of X and Y, also find $P(X<Y)$. (8)

(ii) Suppose the random variables X and Y have the joint p.d.f

$$f(x,y)=\begin{cases} e^{-(x+y)}, & x>0,\ y>0 \\ 0 & ,\ otherwise \end{cases}$$

Find the joint p.d.f of random variables $U=\dfrac{X+Y}{2}$ and $V=Y$ and hence obtain the marginal p.d.f of U. (8)

12. (a) (i) Find the moment generating function of a Poisson distribution. Hence find mean and variance. (8)

(ii) If the probability density of X is given by $f(x)=\begin{cases} 2xe^{-x^2} & for \quad x>0 \\ 0 & elsewhere \end{cases}$ and

Find the p.d.f of the random variable $Y=X^2$ and hence obtain $E(Y)$. (4)

(iii) Let X be a uniform random variable over (-4,4). Find cumulative distribution function of X. And also find $P(|X|>2)$. (4)

(OR)

(b) (i) Find mean, variance and moment generating function of Exponential distribution. (8)

(ii) If X is the number of heads obtained in four tosses of a balanced coin, find probability distribution, mean, variance and moment generating function of X. (8)

13. (a) (i) To determine whether there really is a relationship between an employee's performance in the company 's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtains the results shown in the following table:

Performance in training program

Success in job (employer's rating)

	Below average	average	Above average
poor	23	60	29
average	28	79	60
Very good	9	49	63

Use 0.01 level of significance to test the null hypothesis that performance in the training program and the success in job are independent. (8)

(ii) The following are the number of minutes it took a sample of 15 men and 12 women to complete the application form for a position :

Men:	16.5	20.0	17.0	19.8	18.5	19.2	19.0	18.2	20.8	18.7
	16.7	18.1	17.9	16.4	18.9.					
Women:	18.6	17.8	18.3	16.6	20.5	16.3	19.3	18.4	19.7	18.8
	19.9	17.6.								

Use the U test (Wilcoxon test) at the 0.05 level of significance to test the null hypothesis that the two samples come from identical population against the alternative that two populations are not identical. (8)

(OR)

(b) (i) The following are the Brinell hardness values obtained for samples of two magnesium alloys:

Alloy 1:	66.3	63.5	64.9	61.8	64.3	64.7	65.1	64.5	68.5	63.2
Alloy 2:	71.3	60.4	62.6	63.9	68.8	70.1	64.8	68.9	65.8	66.2 .

Use the0.05 level of significance to test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative hypothesis $\mu_1 - \mu_2 < 0$. (8)

(ii) Test for goodness of fit of a Binomial distribution at the 0.05 level of significance to the following data

X_i:	0	1	2	3
O_i:	1	16	55	228.

(8)

14. a. A lab technician measures the breaking strength of each of 5 kinds of linen thread by means of 4 different instruments, and obtains the following results(in ounces):

	Measuring instrument			
	I_1	I_2	I_3	I_4
Thread1	20.6	20.7	20.0	21.4
Thread2	24.7	26.5	27.1	24.3
Thread3	25.2	23.4	21.6	23.9
Thread4	24.5	21.5	23.6	25.2
Thread5	19.3	21.5	22.2	20.6

Looking upon the threads as treatments and the instruments as blocks, perform an analysis of variance at the level of significance $\alpha = 0.01$. (16)

(OR)

b. The following are the numbers of mistakes made in five successive days by four technicians working for a photographic laboratory.

Technician I:	6	14	10	8	11.
Technician II:	14	9	12	10	14.
Technician III:	10	12	7	15	11.
Technician IV:	9	12	8	10	11.

Test at the 0.05 level of significance whether the differences among the four sample means can be attributed to chance. (16)

15.(a) (i) Construct $\overline{X}$-chart for following data

Sample Number	Observations			Sample Number	Observations		
1	32,	36,	42	5	42,	45,	34
2	28,	32,	40	6	50,	29,	21
3	39,	52,	28	7	44,	52,	35
4	50,	42,	31	8	22,	35,	44

Also determine whether the process is in control. (8)

(ii)The following data gives the number of defectives in 10 samples, each of size 100.

Construct a np-chart for these data and Also determine whether the process is in control.

Sample Number	1	2	3	4	5	6	7	8	9	10
Number of defectives	24	38	62	34	26	36	38	52	33	44

(8)

(OR)

(b) (i) Construct R-chart for following data:

Sample Number	Observations			
1	1.7	2.2	1.9	1.2
2	0.8	1.5	2.1	0.9
3	1.0	1.4	1.0	1.3
4	0.4	-0.6	0.7	0.2
5	1.4	2.3	2.8	2.7
6	1.8	2.0	1.1	0.1
7	1.6	1.0	1.5	2.0
8	2.5	1.6	1.8	1.2
9	2.9	2.0	0.5	2.2

Comment on state of control. (8)

(ii) Construct a control chart for fraction defectives (P-Chart) for following data:

Sample Number	1	2	3	4	5	6	7	8	9	10
Sample Size	90	65	85	70	80	80	70	95	90	75
No of Defectives	9	7	3	2	9	5	3	9	6	7

(8)

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