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B.E. DEGREE END SEMESTER EXAMINATIONS, MAY 2013

COMMON TO MECHANICAL, MATERIAL SCIENCE AND MANUFACTURING ENGINEERING BRANCH

SIXTH SEMESTER - (REGULATIONS 2008)

ME9351 FINITE ELEMENT ANALYSIS

Time: 3 Hours Max. Marks: 100

Answer all Questions

PART-A (10 X 2 = 20 Marks)

1 Give one example each for Boundary, Initial and Eigen Value Problems.

2 Write the Governing equation for 1D heat transfer through a fin with conduction and convection and the associated possible boundary conditions. Write the weak form of the above equation.

3 Distinguish between shape functions and displacement models with suitable examples and plots.

4 An overhanging shaft carries a rotor at the end. Plot the possible mode shapes if the shaft is housed in double row antifriction bearing.

5 What is meant by a CST element? Why is it called so?

6 With suitable examples and the governing equations distinguish between vector and scalar variable problems.

7 Derive the [B] (Strain displacement) matrix for a bi-linear rectangular element.

8 What are the 2D approximations of 3D problems? Give suitable examples for each.

9 What are serendipity elements? Sketch a few such 2D and 3D elements.

10 Write down the shape functions for a linear isoparametric hexahedral element.

PART-B (5 X 16 = 80 Marks)

11 A ride in a theme park consists of a link which rotates about a pivot at 6 rpm as shown in Fig.11. It carries a cage which houses about 10 passengers. The total weight of the cage and passengers is 20kN. The link is 5m long and has a uniform cross sectional area of rectangular cross section of dimensions 15cm by 10 cm. It is desired to check for any elongation of the link. E= 200GPa, $\gamma = 0.0785 N/cm^3$

i) How will you mathematically model the problem?

ii) Determine using any weighted residual technique or the Ritz technique, the variation of displacement along a link and hence the displacement at the tip.

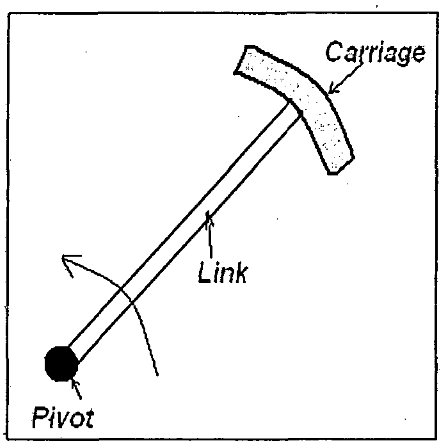


Fig.11

12 a) A metallic fin of square cross section of side 5cm and 30cm long is attached to a wall whose temperature is 100° C. If the thermal conductivity of the material of the fin is 37 W/m°C and convection heat transfer coefficient is 10 $W/m^2 °C$, determine, using 3 elements, the temperature distribution assuming that the tip of the fin is open to the atmosphere and that the ambient temperature is 40° C. Tabulate the temperatures at every 10 cm from the wall and determine the temperature at a distance of 13 cm from the wall

(OR)

12 b) Compute the reactions for the beam loaded as shown in Fig. 12.b. E = 200 GPa, ρ=0.78x10^6 kg/m^3 and the beam is of rectangular cross section of depth 0.6m and width 0.4 m . Determine also the natural frequency of the beam.

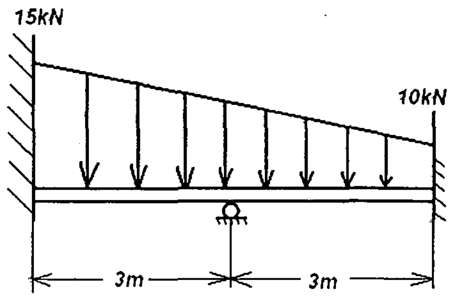


Fig.12.b

13 a) i) For the square shaft of cross section 1cm x 1 cm as shown in Fig.13.a it was decided to determine the stress distribution using FEM by solving for the stress function values. Considering geometric and boundary condition symmetry 1/8 th of the cross section was modeled using two triangular elements and one bilinear rectangular element as shown. The element matrices are given below. Carry out the assembly and solve for the unknown stress function values and hence determine the shear stress distribution. The element connectivity is 1-2-4 for triangular element and 2-3-5-4 for rectangular element. (12 marks)

For Triangle:

$$K = 1/2 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad r = \begin{bmatrix} 29.1 \\ 29.1 \\ 29.1 \end{bmatrix}$$

For Rectangle

$$K = 1/6 \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \qquad r = \begin{bmatrix} 43.6 \\ 43.6 \\ 43.6 \\ 43.6 \end{bmatrix}$$

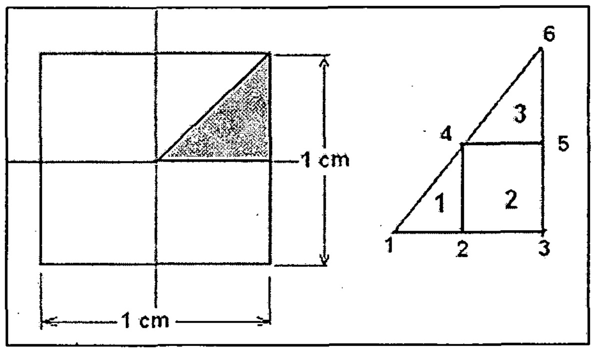


Fig. 13.a

ii) Linear triangular elements and a linear quadrilateral element could be used for discretising a plate fin attached to a transformer and open to convection and conduction. Which element would be better for this application? Write down the shape functions for each and plot the variation of the same. (4marks)

(OR)

13.b) i) A bilinear rectangular element has coordinates as shown in Fig. 13.b and the nodal temperatures are T_1=120° C, T_2=80° C, T_3=50° C, T_4=90° C. Compute the temperature at the point whose coordinates are (2.5,2.5). Also determine the 96° C isotherm. Comment on the Jacobian matrix evaluated at the centroid for this element if it were transformed to Natural coordinates. (10 Marks)

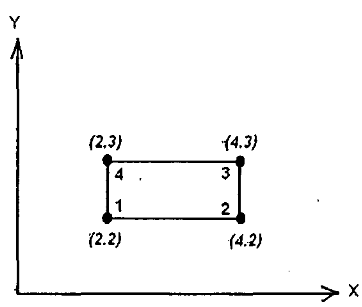


Fig.13.b

ii)) Derive the shape functions for a corner node and a mid side node for a quadratic triangular element in natural coordinates and plot the variation of the same. (6 marks)

14 a) The thin plate subjected to in plane loading as shown in Fig14.a is discretised into two linear triangular elements. Determine the nodal loads, the strain displacement matrix and the constitutive matrix. How are the stiffness matrix and elemental stresses and strains determined? The plate thickness is 10mm, E = 70 GPa and μ=0.3. (12 marks)

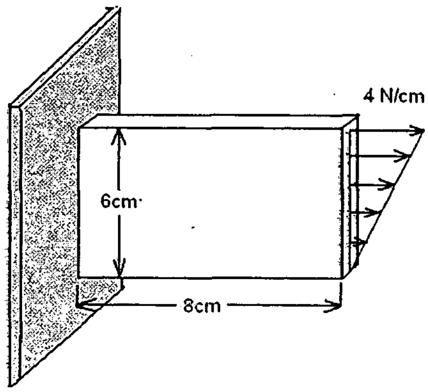


Fig.14.a

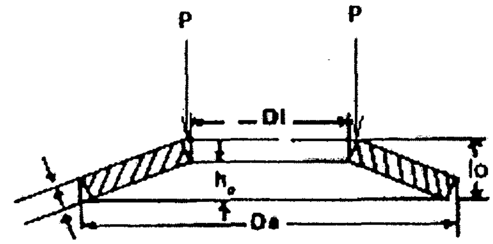


Fig.14.b

ii) Derive the stiffness matrix for a ID linear isoparametric element. (4 marks)

(OR)

14.b) i) Give the Strain displacement relations for axisymmetric analysis and hence derive the Strain displacement matrix. Give at least two practical examples where you could go in for axisymmetric analysis and discuss how the member is discretised. (10 marks)

ii) The Belleville Spring Shown in Fig.14.b needs to be analysed to check for its design. Discuss about the modeling procedure, choice of element, meshing criterion and boundary conditions to be used in Finite element Analysis. (6marks)

15 a) i) Using Gauss Quadrature evaluate the following integral (8 marks)

$$I=\int_{-1}^{+1}\int_{-1}^{+1}\frac{(2+\xi+\xi^2)}{(3+2\eta+\eta^2)}d\xi\, d\eta$$

ii) Evaluate the shape functions for a corner node and mid side node of a quadratic quadrilateral serendipity element and plot its variation. What are the advantages of this element? (8 marks)

(OR)

15 b) i) Explain the concept behind and advantages of natural coordinate transformation. Differentiate between subparametric, isoparametric and superparametric elements. (8 marks)

ii). For the four noded element shown in Fig15.b determine the Jacobian and evaluate its value at the point (1/3, 1/3). (8marks)

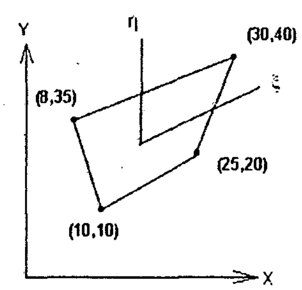


Fig.15.b

$$StiffnessMatrix[K]^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\{f\}^e = \frac{ql}{2}\begin{Bmatrix} 1 \\ l/6 \\ 1 \\ -l/6 \end{Bmatrix}$$

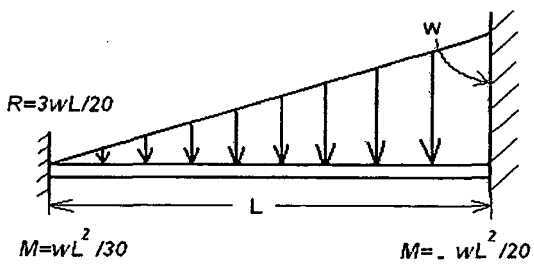
$$[M] = \frac{\rho A \ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix}$$

$$N_1 = 1 - \left(\frac{3x^2}{l^2}\right) + \left(\frac{2x^3}{l^3}\right)$$

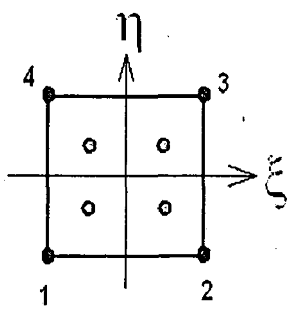
$$N_2 = x - \left(\frac{2x^2}{l}\right) + \left(\frac{x^3}{l^2}\right)$$

$$N_3 = \left(\frac{3x^2}{l^2}\right) - \left(\frac{2x^3}{l^3}\right)$$

$$N_4 = -\left(\frac{x^2}{l}\right) + \left(\frac{x^3}{l^2}\right)$$



No.of points	Location	Weight W_i
1	ξ_1 = 0.00000	2,00000
2	ξ_1 , ξ_2 ± 0.57735	1.00000
3	ξ_1, ξ_3 = ± 0.77459 ξ_2 = 0.00000	0.55555 0.88888



$$\int_{-1}^{+1}\int_{-1}^{+1} F(\xi,\eta) = \Sigma\Sigma\, f\xi_i\, \eta_i) w_i w_j$$

$$\xi, \eta = \pm 0.57735$$