Roll No.

B.E / B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY 2013

Common to Bio-medical and Electronics and Communication Engineering

Semester IV

MA 9263 Probability and Random Processes

(Regulation 2008)

(Statistical tables are permitted in the Examination Hall)

Time: 3 Hours — Answer ALL Questions — Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. Let X be a random variable taking values -1, 0 and +1 such that P{X= -1) = 2P{X=0} = P{X=1}. Find the mean of 3X.

2. Find a constant b>0 so that the function $f_X(x) = \frac{e^{3x}}{4}, 0 \le x \le b$ is a valid probability density function.

3. Let X and Y be continuous random variables with joint probability density function $f_{XY}(x,y) = \frac{3}{2}\left(x^2+y^2\right), 0<x<1,\ 0<y<1$, find $f_{X|Y}(x|y)$.

4. If X=4Y+5 and Y=kX+4 are the lines of regression of X on Y and Y on X respectively, show that 0<4k<1.

5. Telephone calls arrive at a switching center in a Poisson manner at an average rate of 75 calls per minute. What is the probability that more than 3 calls arrive within a 5-second period.

6. If x(t) is wide-sense stationary process with autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha|\tau|}$, determine the second-order moment of the random variable x(8)-x(5).

7. Show that $R_{XX}(0) \ge R_{XX}(\tau)$.

8. The power spectral density of a stationary random process is a constant in a symmetrical interval about zero and zero outside the interval. Compute the autocorrelation function.

9. Assume that the input x(t) to a linear time-invariant system is white noise. What is the power spectral density of the output process y(t) if the system response H(ω) is given by $H(\omega) = \begin{cases} 1 & \omega_1 < |\omega| < \omega_2 \\ 0 & \text{otherwise} \end{cases}$.

10. A linear system has the impulse response $h(t) = e^{-at}$, t≥0 and a>0. Find the power transfer function of the system.

Part – B (5 x 16 = 80 marks)

11. (i)A wide-sense stationary process x(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-7t}$, t≥0. If the autocorrelation function of the process is $R_{xx}(\tau) = e^{-4|\tau|}$ and the output process is y(t),find $S_{yy}(\omega), S_{xy}(\omega)$ and $R_{xy}(\tau)$ (10)

(ii)A linear system has a transfer function $H(\omega) = \frac{\omega}{\omega^2 + 15\omega + 50}$. Determine $S_{yy}(\omega)$ when the input function is a stationary random process x(t) with $R_{xx}(\tau) = 10e^{-|\tau|}$ (6)

12. a) (i)A random variable X has the moment generating function given by $M_X(t) = \left(\frac{4}{10}e^t + \frac{6}{10}\right)^{15}$. Find the standard derivation of X. (6)

(ii)The random variable X which denotes the interval between two consecutive events has the probability density function, $f_X(x) = 4x^2e^{-2x},\ x \ge 0$. If we assume that intervals between events are independent, determine E[X], and $P\{X \le 6\}$. (6)

(iii)Let X be a random variable with the probability density function $f_X(x) = 2e^{-2x}$, x>0. Find the probability density function of $Y = \sqrt{X}$ (4)

OR

b) (i) Find the moment generating function of an Exponential random variable X with parameter λ. Obtain the standard derivation of X. (6)

(ii) 20% of the population of a particular city wear glasses. If you randomly stop people from that city, determine the probability that

1. It takes **exactly** 10 tries to get a person who wears glasses.
2. It takes **atleast** 10 tries to get a person who wears glasses. (6)

(iii) Let X be a continuous random variable with the density function $f_X(x) = \frac{1}{4}$, -2<x<2. Find the probability density function of $Y = X^4$. (4)

13. a) (i) Let X and Y be independent(strictly positive) exponential random variables each with parameter λ. Are the random variables X+Y and $\frac{X}{Y}$ independent? (8)

(ii) The random variable X is Uniformly distributed between 1 and 2. If S is the sum of 40 independent experimental values of X, evaluate $P\{55 \le S \le 65\}$. (8)

OR

b) Let the joint probability density function of (X,Y) be $f_{XY}(x,y) = e^{-y},\ 0<x<y<\infty$ Find the correlation coefficient r(X,Y) (16)

14\. a) (i) Classify the random process $x(t) = A\cos\Omega t$ where A and Ω are random variables with joint probability density function $f_{A\Omega}(\alpha,\beta) = \frac{1}{8}, 0 < \alpha < 2, 8 < \beta < 12$. (8)

(ii) x(t) is a random telegraph type process composed of pulses of heights +1 and -1 respectively. The number of transactions of the t-axis in a time 2 is $P(k\ transitions) = \frac{e^{-4}4^K}{K!}$. k=0,1,... Classify the above process (8)

OR

b) (i) Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Suppose each arrival is registered with probability p independent of other arrivals. Let $\{y(t), t \geq 0\}$ be the process of registered arrivals. Prove that y(t) is a Poisson process with parameter λp. (8)

(ii) Show that the random process $x(t) = \cos(t + \Phi)$ where Φ is uniformly distributed in $(0, 2\pi)$ is 1. first order stationary 2. wide-sense stationary (8)

15\. a) (i) Two random processes x(t) and y(t) are given by $x(t) = A\cos(\omega t + \theta)$ and $y(t) = A\sin(\omega t + \theta)$ where A and ω are constants and θ is a uniform random variable over $(0, 2\pi)$. Find the cross-spectral density functions $S_{xy}(\omega)$ and $S_{yx}(\omega)$ and verify $S_{xy}(\omega) = S_{yx}(-\omega)$ (8)

(ii) Given the power spectral density $S_{xx}(w) = \frac{9}{w^2 + 64}$ of a random process x(t), find the autocorrelation function of y(t) and the average power in the process (8)

OR

b) (i) Given x(t)=3 cos(ω_ct+Φ) and y(t) = 2 cos(ω_ct+Θ) where Φ and Θ are RVs with uniform density functions over the period $T = \frac{2\pi}{\omega_c}$ but $\Theta = \Phi - \frac{\pi}{2}$, find $R_{XY}(\tau)$ and $R_{YX}(\tau)$ (8)

(ii) The autocorrelation function of a signal is $e^{-\frac{\tau^2}{2k^2}}$ where k is a constant. Find the power spectral density and average power of the signal (8)

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