27|4|17

Reg. No. | | | | | | | | | | |

**B.E./ B. Tech (Full Time) DEGREE END SEMESTER EXAMINATION, April / May - 2013**

**THIRD SEMESTER**

**MA 9211 - MATHEMATICS III (Regulation - 2008)**

**COMMON TO ALL BRANCHES**

**Time: 3 hours** **Maximum: 100 Marks**

**Answer ALL Questions**

**Part – A $(10\times 2 = 20$ marks)**

1. State Dirichlet's conditions for existence of Fourier series.
2. Define the complex form Fourier series of $f(x)$, in $(c, c+2l)$.
3. Obtain the partial differential equation by eliminating the arbitrary constants $a$ and $b$ from $z = (x^2 + a)(y^2 - b)$.
4. Find the complete solution of $pq = 1$.
5. What are the possible solutions of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$?
6. What is meant by steady state?
7. If the Fourier transform of $f(x)$ is $F(s)$, then find the Fourier transform $e^{iax} f(x)$.
8. Show that $F_S\{xf(x)\} = -\frac{d}{ds}[F_C(s)]$, where $F_S$ and $F_C$ are Fourier sine and cosine transforms respectively.
9. Find the Z-transform of $n\,2^n$.
10. State initial value theorem for Z-transform.

**Part – B $(5\times 16 = 80$ marks)**

11. i) Find the Z-transform of $\frac{1}{n(n+1)}$, for $n \geq 1$. (8)

    ii) Using Z-transform, solve $u_{n+2} + 3u_{n+1} + 2u_n = 0$, given $u_0 = 1, u_1 = 2$. (8)

12. (a) i) Find the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ of periodicity $2\pi$. (8)

    ii) Obtain half range Fourier sine series expansion of $f(x) = x$ in $0 < x < l$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (8)

    (OR)

    (b) i) Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & if\ -\pi < x < 0 \\ x, & if\ \ 0 < x < \pi \end{cases}$. (8)

    ii) Obtain half range Fourier cosine series expansion of $f(x) = (x-1)^2$ in $0 < x < 1$. (8)

13. (a) i) Find the singular solution of $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)

ii) Find the general solution of $(D^2 - 2DD' + D'^2)z = \cos(x - 3y) + e^{-2x}$. (8)

(OR)

(b) i) Find the general solution of $(y - z)p + (z - x)q = (x - y)$. (8)

ii) Find the complete solution of $p^2 + q^2 = x + y$. (8)

14. (a) Solve the problem of the vibrating string for the following boundary conditions:

(i) $y(0,t) = 0,\ for\ t \geq 0$, (ii) $y(l,t) = 0,\ for\ t \geq 0$,

(iii) $y(x,0) = 3(lx - x^2),\ for\ 0 \leq x \leq l$ (iv) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0,\ for\ 0 \leq x \leq l$. (16)

(OR)

(b) A bar 30cm, long, with insulated lateral surface, has its ends A and B kept at $20^\circ$ and $80^\circ$, respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^\circ$C. and kept so. Find the resulting temperature $u(x,t)$ of the rod by taking $x = 0$ at A. (16)

15. (a) i) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & if\ |x| < 1 \\ 0, & otherwise \end{cases}$. (8)

ii) Find the Fourier cosine transform of $e^{-ax}, a > 0$, hence show that $\int_0^\infty \frac{\cos sx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-as}$. (8)

(OR)

(b) i) Obtain the Fourier transform of $f(x) = e^{-a^2 x^2}$, for $a > 0$, hence find the Fourier transform of $e^{-\frac{x^2}{2}}$. (8)

ii) By obtaining the Fourier sine transform of $e^{-ax}$ ($a > 0$), evaluate $\int_0^\infty \frac{x^2 dx}{(x^2 + 16)^2}$. (8)

* * *