Roll No.

B.E (FT) END SEMESTER EXAMINATIONS – APRIL 2019

Computer Science and Engineering

Second Semester

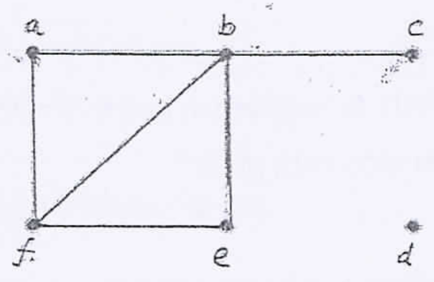
MA6251 DISCRETE MATHEMATICS

(Regulation 2018 - RUSA)

Time: 3 Hours Answer ALL Questions Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. State the converse, contrapositive, and inverse of the following conditional statement "I come to class whenever there is going to be a quiz".
2. What is wrong with the proof of Theorem that " if n^2 is positive, then n is positive"?
3. Use mathematical induction to prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.
4. What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots 1, 1, 1, 1, −2, −2, −2, 3, 3, −4?
5. Find the number of vertices, the number of edges, and the degree of each vertex in the following undirected graph.



6. Check whether the bipartite graph $K_{2,4}$ is Eulerian or Hamiltonian. Justify the claim.

7. State the relationship between semigroup and monoid. Give an example. Justify the claim.

8. When a ring is a field and also an integral domain? State the relationship between field and integral domain.

9. Let S = {a, b, c}. Draw the Hasse diagram of a poset $(\rho(S), \subseteq)$.

10. Give an example of a lattice which is a complemented lattice but not distributive lattice.

PART – B (8 x 8 = 64 marks)
(Answer any 8 questions)

11. Check whether conditional statement $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$ is a tautology or contradiction.

12. Obtain the principal disjunctive normal form of $([(\neg Q \vee \neg R) \rightarrow \neg P] \wedge (Q \vee R)) \rightarrow P$ by using equivalences.

13. Verify the validity of the arguments: "One student in this class knows how to write programs in JAVA". "Everyone who knows how to write programs in JAVA can get a high-paying job".Therefore, "Someone in this class can get a high-paying job".

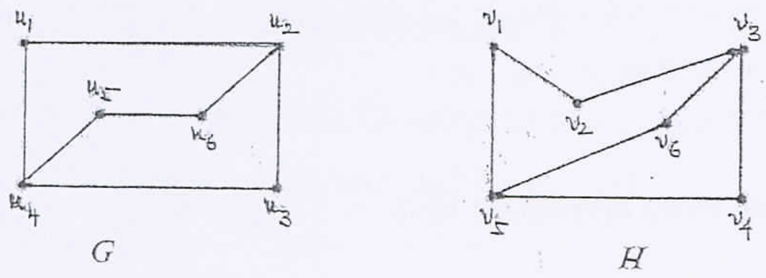
14.(i) How many positive integers between 100 and 999 inclusive are not divisible by either 3 or 4? (4)

(ii) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? (4)

15. Find the generating function and hence solve the recurrence relation

$$a_k = 4a_{k-1} + 3k.2^k, k \geq 1 \text{ with initial condition } a_0 = 4.$$

16. Determine whether the graphs G and H given below are isomorphic.



17. If G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then show that G has a Hamilton circuit.

18. Let S_3 denote the set of all permutations of a set $S = \{1, 2, 3\}$. Show that $(S_3, \lozenge)$ is a permutation group under a binary operation $\lozenge$, (right composition of permutations). Is it an abelian? Justify the claim.

19. Show that if H is a subgroup of a finite group $\langle G, * \rangle$, then $O(H)$ divides $O(G)$.

20. Let $\langle G,* \rangle$ and $\langle H, \Delta \rangle$ be any two groups, and $f: G \rightarrow H$ be a group homomorphism. Show that the $Ker(f)$ is a normal subgroup of G.

21. Let a, b and c be arbitrary elements in a lattice $\langle L, \leq \rangle$. Prove the following inequalities:

(i) when $b \leq c$ then $a \wedge c \leq a \wedge c$ and $a \vee b \leq a \vee c$;

(ii) when $a \leq c$, then $a \vee (b \wedge c) \leq (a \vee b) \wedge c$.

22. Show that De Morgan's laws hold in a Boolean algebra.

PART – C (2 x 8 = 16 marks)

23. Determine how many integer solutions that are to $x_1 + x_2 + x_3 + x_4 = 19$, if $0 \le x_i < 8$ for all $1 \le i \le 4$.

24. (i) Show that every distributive lattice is modular. Is the converse true? Justify the claim. (4)

(ii) If $\langle L, \wedge, \vee, 0, 1 \rangle$ is a distributive lattice, then show that each element $x \in L$ has at most one complement. (4)

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