25/4/19

Regn. No:

B.E / B.Tech (Full Time) Degree End Semester Examinations - April 2019
Electronic and Communication Engineering Branch
IV Semester B.E (ECE)
MA7353 Linear Algebra and Numerical Methods
(Regulation 2015)

Duration: 3 Hours

Total marks= 100

Show all numerical computations for five decimal places.

Part A

(10 x 2 = 20 Marks)

1. Examine whether or not $W = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 + 2a_2 - a_3 = 0\}$ is subspace of $\mathbb{R}^3$.
2. If x, y and z are vectors in a vector space V such that $x + z = y + z$, then show that $x = y$.
3. Write down the matrix form of the linear transformation that is described by a rotation about an angle θ in the counter clockwise direction in xy-plane.
4. Does there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ for which $T(1,2) = (7,9)$ and $T(3,6) = (14,27)$? Justify your answer.
5. If $\langle x, y\rangle = \langle x, z\rangle$ for every x in an inner product space V, then show that $y = z$.
6. What is the orthogonal complement $S^{\perp}$ of $S = \{(1,1)\}$ in $\mathbb{R}^2$? Describe it geometrically.
7. What is the drawback in Gauss-Elimination Method? How is it rctified? Explain.
8. Compare Gauss-Jacobi and Gauss-Seidel methods for solving linear system of equations of the form $A\overline{x} = \overline{b}$.
9. Show that if λ is the smallest non-zero eigenvalue of an invertible matrix A, then $\frac{1}{\lambda}$ is the largest eigen value of A^{-1}
10. Define a singular value of a matrix.

Part B

(5 X 16=80 Marks)

11.a(i) Using Gram-Schmidt orthogonalization process construct an orthogonal set from the given set $S = \{(1,0,1,0), (1,1,1,1), (0,1,2,1)\}$ of $\mathbb{R}^4$. (8)

(ii) Using Least square approximation determine the best linear fit for the data: $\{(1,2), (2,3), (3,5), (4,7)\}$. (8)

12.a(i) Let V be the set of of all polynomials of degree less than or equal to 2 with real coefficients. Show that V is a vector space over $\mathbb{R}$ with respect to usual addition of polynomials and usual multiplication of a polynomial by a real number. Verify all the conditions of a vector space. (8)

(ii) Let $S = \{\bar{v}_1 = (1,-1,2), \bar{v}_2 = (1,-2,1), \bar{v}_3 = (1,1,4)\}$ be a subset of $\mathbb{R}^3$. Determine whether S is linearly independent or linearly dependent set in $\mathbb{R}^3$. If S is linearly dependent express $\bar{v}_3$ in terms of $\bar{v}_1$ and $\bar{v}_2$, otherwise find the linear span $L(S)$. (8)

(OR)

12.b(i) Let $S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}$ be a subset of $\mathbb{M}_{2X2}(\mathbb{R})$. Determine whether S is linearly independent or linearly dependent set in $\mathbb{M}_{2X2}(\mathbb{R})$. Further, find the linear span $L(S)$. Is S a basis of $\mathbb{M}_{2X2}(\mathbb{R})$.? (8)

(ii) Determine whether or not the set $S = \{1 - 2x - 2x^2, -2 + 3x - x^2, 1 - x + 6x^2\}$ forms a basis for $\mathbb{P}_2(\mathbb{R})$. (8)

13.a(i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (x + 3y, 0, 2x - 4y)$. Compute the matrix of the transformation with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. Find $N(T)$ and $R(T)$. Is T one-to-one? Is T onto? Justify your answer. (8)

(ii) Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be defined by $T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$. Find bases for $N(T)$ and $R(T)$ and hence verify the dimension theorem. Is T one-to-one and onto? Justify your answer. (8)

(OR)

13.b(i) For the linear operator $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ defined as $T(f(x)) = f(x) + xf'(x) + f''(x)$, find the eigenvalues of T and an ordered basis $\mathcal{B}$ for $P_2(\mathbb{R})$ such that the matrix of the given transformation with respect to the new resultant basis $\mathcal{B}$ is a diagonal matrix. (8)

(ii) Solve the system of differential equations using diagonalization and discuss its stability: (8)

$$\begin{aligned} x'(t) &= x(t) + y(t) \\ y'(t) &= 3x(t) + 2y(t). \end{aligned}$$

14.a(i) Solve the following system by Gauss elimination method: (8)

$$\begin{aligned} 2x + 6y + 10z &= 0 \\ x + 3y + 3z &= 2 \\ 3x + 14y + 28z &= -8. \end{aligned}$$

(ii) Write down the Gauss-Seidel iteration scheme for the following system. Then solve the system by the same method for four iterations starting with the initial vector $(1, 2, 2)^T$. (8)

$$\begin{aligned} 4x - y + z &= 7 \\ 4x - 8y + z &= -21 \\ -2x + y + 5z &= 15. \end{aligned}$$

(OR)

14.b(i) Solve the following system by LU decomposition method : (8)

$$\begin{aligned} x + y + z &= 3 \\ 2x - y + 3z &= 16 \\ 3x + y - z &= -3. \end{aligned}$$

(ii) Solve the following system by Gauss-Jordan method: (8)

$$\begin{aligned} 2x - 6y + 8z &= 24 \\ 5x + 4y - 3z &= 2 \\ 3x + y + 2z &= 16. \end{aligned}$$

15.a(i) Obtain by power method the numerically largest eigenvalue and its corresponding eigenvector for the matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{pmatrix}$ starting with the vector $(1, 0, 0)^T$ for three iterations. (8)

(ii) Construct a QR decomposition for the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. (8)

(OR)

15.b(i) Using the Jacobi rotation method, find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$. (8)

(ii) Construct a singular value decomposition for the matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$. (8)

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