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Roll No. | | | | | | | | | | |

B.E / B.Tech (Full -Time) DEGREE END SEMESTER EXAMINATIONS, APRIL 2019
CSE Semester II
MA7355- PROBABILITY AND QUEUEING THEORY
(CBCS Regulation)

Time: 3 Hours | Answer ALL Questions | Max. Marks 100

(Use of Calculators and Statistical Tables is permitted in the exam hall)

PART-A (10 x 2 = 20 Marks)

1. Given *X*, the number of customers entering a post office, during any period of length *t*, to be a random variable, with probability function, $p_X(n) = k\frac{(2t)^n}{n!}, n = 0,1,2,...$ Determine the value of *k*.
2. Let X be a random variable with the density function $f_X(x) = 2e^{-2x}, x > 0$. Find the probability density function (pdf) of $Y = \sqrt{X}$.
3. The random variable (*X*,*Y*) has the pdf $f_{XY}(x,y) = xye^{-y^2/4}$, $0 \le x \le 1$, $y \ge 0$. Find the marginal pdf of *X*.
4. Let X and Y be two independent random variables. Find the value of *Cov(X,XY)*.
5. Customers arrive at a post office at a Poisson rate of 3 per minute. What is the probability that the next customer does not arrive during the next 3 minutes?
6. Given the transition probability matrix (TPM) $P = \begin{pmatrix} .8 & .2 \\ .4 & .6 \end{pmatrix}$ for a Markov chain with state space (1, 2), use the transition probabilities to analyze whether state 1 is recurrent, (without using the transition diagram).
7. Consider a store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that 9 customers arrive on an average every 5 minutes and the cashier can serve 10 in 5 minutes. Calculate the average number of customers queueing for service.
8. Explain the Kendall notation for a finite source Markovian queueing model.
9. State the Pollaczek–Khintchine (P-K) formula for M/D/1:GD/∞/∞ queueing model.
10. Write the model equation for an open queueing network model.

Part – B (5 x 16 = 80 Marks)

11. (i) For a Poisson random variable *X* with parameter *λ*, derive the moment generating function (MGF). Using the MGF obtain its mean and variance. (10)

    (ii) The amount of cereal in a box is Normal with mean 16.5 ounces. If the package is required to fill at least 90% of the cereal boxes with 16 or more ounces of cereal, what is the largest standard deviation for the amount of cereal in a box? (6)

12. a) (i) The joint pdf of a random variable (X,Y) is given by $f_{XY}(x,y) = x + y$, $0 \le x \le 1$, $0 \le y \le 1$. Find the pdf of *U*=XY. Assume that *V*=Y. (8)

(ii) 20 dice are thrown. Find, approximately, using the Central Limit Theorem, the probability that the sum obtained is between 65 and 75. (8)

(OR)

b) Let the joint pdf of the random variable (X,Y) be $f_{XY}(x,y) = \frac{x^3}{2} e^{-x(y+1)}\ x > 0, y > 0.$

Find the correlation coefficient $r(X,Y)$. (16)

13. a) Given the TPM $P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ for a Markov chain $\{X_n, n \geq 0\}$ with state space {1, 2, 3}. Classify the states, analyze the Ergodicity of the Markov chain and hence find the limiting distribution of the Markov chain. (16)

(OR)

b) (i) State the postulates for a Poisson process $\{N(t), t \geq 0\}$ and hence derive that

$$P_n(t) = P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}\ ,\ n = 0,1,2\ldots,\ \lambda > 0.$$ (10)

(ii) Show that a random process which is strict sense stationary of order n is also stationary of all orders lower than n. (6)

14. a) (i) Derive the steady-state probability distribution for a $M/M/C:GD/\infty/\infty$ model. (8)

(ii) Consider a single server queueing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is 2. Find the expected number in the system and its average waiting time. (8)

(OR)

b) (i) Derive the steady-state probability distribution for a $M/M/1:GD/N/\infty$ model. (8)

(ii) A 3 stall automobile inspection facility can accommodate a maximum of 7 cars. The arrival pattern is Poisson with a mean of one car every minute. The service time is exponential with a mean of 6 minutes. Find the average number of customers in the system and its average waiting time. (8)

15. a) (i) For the $M/G/1:GD/\infty/\infty$ model, derive the Pollaczek–Khintchine formula – (10)

(ii) A patient goes to a single doctor clinic for a general check up and has to go through four phases. The doctor takes on an average 4 minutes for each phase of the check up and the time taken for each phase is exponentially distributed. If the arrival of the patients at the clinic are Poisson at an average rate of 3 per hour, what is the average time spent by a patient in the examination and in the clinic. (6)

(OR)

b) Consider a Jackson network with three service facilities that have the parameters as shown in the following table.

Facility j	c_j	μ_j	r_j	p_{ij}		
				i=1	i=2	i=3
j=1	1	40	10	0	0.3	0.4
j=2	1	50	15	0.5	0	0.5
j=3	1	30	3	0.3	0.2	0

Find the steady-state distribution of the number of customers at facilities 1, 2 and 3 respectively. Then show the product form solution for the joint distribution of the number at the respective facilities. What is the probability that all the facilities have empty queues (no customers waiting to begin service)? Find the expected total number of customers in the system. Also find the expected total waiting time (including the service times) for a customer. (16)