Reg.No. | | | | | | | | | | |

B.E [Full Time] DEGREE END SEMESTER EXAMINATION, April / May - 2019

MECHANICAL ENGINEERING

THIRD SEMESTER

MA8302 – PARTIAL DIFFERENTIAL EQUATIONS (Regulation – 2012)

Time: 3 hours    Answer ALL Questions    Maximum: 100 Marks

PART-A (10 x 2 = 20 Marks)

1. Form the partial differential equation by eliminating the arbitrary constants *a* and *b* from $z^2 = a^2x^2 + b^2y^2$.
2. Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = 0$.
3. Write complex form of the Fourier series in the interval $0 < x < 2l$
4. Find the root mean square value of $f(x) = 1 - x$ in the interval $0 < x < 1$
5. What is the constant $c^2$ in the wave equation $u_{tt} = c^2 u_{xx}$?
6. Write all the possible solutions of the two dimensional heat flow equation?
7. Define tri-diagonal matrix.
8. Why the Crank Nicholson's scheme is called as an implicit scheme.
9. Compare Gauss Jacobi and Gauss Seidel methods.
10. Write the diagonal five point formula and standard five point formula.

Part – B ( 5 x 16 = 80 marks)
(Question No.11 is Compulsory)

11. (i) Solve the following system of linear equations by using Gauss-Elimination method: $x + 2y + 3z - u = 10$, $2x + 3y - 3z - u = 1$, $2x - y + 2z + 3u = 7$, $3x + 2y - 4z + 3u = 2$. (8)

(ii) Solve $u_{xx} = u_t$ under the conditions $u(0,t) = 0$, $u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$ using Bender-Schmidt method (Take $h = 0.2, \lambda = 1/2$.) (8)

12. a) (i) Form the partial differential equation by eliminating the arbitrary functions from $xyz = f(x^2 + y^2 - z^2)$. (8)

(ii) Solve $x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$. (8)

(OR)

b) (i) Solve $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Find the general solution of $(D^2 - DD' - 20D'^2)z = e^{5x+y} + \sin(4x - y)$. (8)

ADDITIONAL CONTROLLER OF EXAMINATIONS AU CHENNAI 600 025

13\. a) (i) Obtain the Fourier series expansion of $f(x)=\begin{cases} x, & 0<x\le\pi \\ 2\pi-x, & \pi\le x<2\pi \end{cases}$ and (8)

Hence find the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\dots\dots$

(ii) Find the Fourier series expansion of $f(x)=x^2-2$ in $-2<x<2$. (8)

(OR)

b) (i) Find the half range cosine series expansion of $f(x)=x(\pi-x)$ in $0<x<\pi$. (8)

Hence find the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}$.

(ii) Find the Fourier sine series upto third harmonic for the function $f(x)$ from the table: (8)

| x | 0 | $\pi/6$ | $2\pi/6$ | $3\pi/6$ | $4\pi/6$ | $5\pi/6$ | $\pi$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.34 | 2.2 | 1.6 | 0.83 | 0.51 | 0.88 | 2.34 |

14\. a) A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities $v=\begin{cases} \frac{cx}{l} & ,0<x<l \\ \frac{c(2l-x)}{l} & ,l<x<2l \end{cases}$, $x$ being the distance from an end point. Find the displacement of the string. (16)

(OR)

b) A bar $l$ cm long with insulated sides has its ends A and B kept at $30^0C$ and $80^0C$ respectively until steady state condition prevail. The temperature at A is then suddenly increased to $40^0C$ and at the same instant that at B is reduced to $60^0C$. Find subsequent temperature at any point of the bar at any time. (16)

15\. a) (i) Solve the Poisson's equation $\nabla^2u=-10(x^2+y^2+10)$ over the square mesh with sides $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary with 1 unit length on the mesh. (8)

(ii) Solve the wave equation $u_{xx}=u_{tt}$ with the boundary conditions $u(0,t)=0$, $u(1,t)=0$, $u(x,0)=10+x(1-x)$ and $u_t(x,0)=0$ by taking $h=0.1$ (upto $t=0.5$). (8)

(OR)

b) (i) Solve the following system of linear equations by Gauss-Seidel method correct to three decimal places: $20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$. (8)

(ii) Solve $u_{xx}+u_{yy}=0$ over the square mesh with 1 unit of sub-square of side 4 units, satisfying the following boundary conditions correct to 2 decimal places: (8)

$u(0,y)=0$ , $0\le y\le 4$; $u(4,y)=8+2y$ , $0\le y\le 4$;

$u(x,0)=\frac{x^2}{2}$, $0\le x\le 4$; $u(x,4)=x^2$ , $0\le x\le 4$.

END