Regn. No: | | | | | | | | | | |

B.E / B.Tech (Full Time) Degree End Semester Examinations - April 2019
Electronic and Communication Engineering Branch
IV Semester B.E (ECE)
MA8401 Linear Algebra and Numerical Methods
(Regulation 2012)

Duration: 3 Hours | Total marks= 100

Show all numerical computations for five decimal places.

Part A (10 x 2 = 20 Marks)

1. Examine whether or not $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ is subspace of $\mathbb{R}^2$.
2. If x, y and z are vectors in a vector space V such that $x + z = y + z$, then show that $x = y$.
3. What is the general form any linear transformation from $\mathbb{R}^3$ into $\mathbb{R}$ with respect to standard bases? Justify your answer.
4. Does there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ for which $T(1, 2) = (7, 9)$ and $T(3, 6) = (14, 27)$? Justify your answer.
5. Let v_1 and v_2 be two nonzero vectors in an inner product space V such that $v_2 = 5v_1$. Examine whether they are linearly independent and also examine whether they are orthogonal.
6. What is the orthogonal complement $S^{\perp}$ of $S = \{(1, -1)\}$ in $\mathbb{R}^2$? Describe it geometrically.
7. Examine whether the following system is diagonally dominant. If it is not diagonally dominant, convert it into a diagonally dominant system.

$$\begin{aligned} x - y + 3z &= 1 \\ 5x - 2y + z &= 6 \\ 2x + 6y - z &= 2 \end{aligned}$$

8. Compare Gauss-Jacobi and Gauss-Seidel methods for solving linear system of equations of the form $A\overline{x} = \overline{b}$.
9. Define an orthogonal matrix and examine whether or not $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an orthogonal matrix.
10. Define a singular value of a matrix.

Part B (5 X 16=80 Marks)

11.a(i) Using Gram-Schmidt orthogonalization process construct an orthogonal set from the given set $S = \{(1, 0, 1, 0), (1, 1, 1, 1), (0, 1, 2, 1)\}$ of $\mathbb{R}^4$. (8)

(ii) Using Least square approximation determine the best linear fit for the data: $\{(1, 2), (3, 4), (5, 7), (7, 9), (9, 12)\}$. (8)

12.a(i) Let V be the set of of all 2×2 matrices with real entries. Show that V is a vector space over $\mathbb{R}$ with respect to addition of matrices and usual multiplication of a matrix by a real number. Verify all the conditions of a vector space. (8)

(ii) Let $S = \{\bar{v}_1 = (1, -1, 2), \bar{v}_2 = (1, -2, 1), \bar{v}_3 = (1, 1, 4)\}$ be a subset of $\mathbb{R}^3$. Determine whether S is linearly independent or linearly dependent set in $\mathbb{R}^3$. If S is linearly dependent express $\bar{v}_3$ in terms of $\bar{v}_1$ and $\bar{v}_2$, otherwise find the linear span $L(S)$. (8)

(OR)

12.b(i) Let $\mathcal{B} = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n\}$ be a subset of a vector space V. Then $\mathcal{B}$ is a basis if and only if each $\bar{v} \in V$ can uniquely be expressed as a linear combination of vectors of $\mathcal{B}$. (8)

(ii) Determine whether or not the set $S = \{1 - 2x - 2x^2, -2 + 3x - x^2, 1 - x + 6x^2\}$ forms a basis for $\mathbb{P}_2(\mathbb{R})$. (8)

13.a(i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (x + y, 0, 2x - y)$. Compute the matrix of the transformation with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. Find $N(T)$ and $R(T)$. Is T one-to-one? Is T onto? Justify your answer. (8)

(ii) Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$ be defined by $T(f(x)) = f'(x) + \int_0^x f(t)dt$. Find bases for $N(T)$ and $R(T)$ and hence verify the dimension theorem. Is T one-to-one and onto? Justify your answer. (8)

(OR)

13.b(i) For the linear operator $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ defined as $T(f(x)) = f(x) + xf'(x) + f''(x)$, find the eigenvalues of T and an ordered basis $\mathcal{B}$ for $P_2(\mathbb{R})$ such that the matrix of the given transformation with respect to the new resultant basis $\mathcal{B}$ is a diagonal matrix. (8)

(ii) Solve the system of differential equations using diagonalization and discuss its stability: (8)

$$\begin{aligned} x'(t) &= 4x(t) + y(t) \\ y'(t) &= 3x(t) + 2y(t). \end{aligned}$$

14.a(i) Solve the following system by Gauss elimination method: (8)

$$\begin{aligned} 2x + 6y + 10z &= 0 \\ x + 3y + 3z &= 2 \\ 3x + 14y + 28z &= -8. \end{aligned}$$

(ii) Write down the Gauss-Seidel iteration scheme for the following system. Then solve the system by the same method for four iterations starting with the initial vector $(0, 0, 0)^T$. (8)

$$\begin{aligned} 20x + y - 2z &= 17 \\ 3 + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}$$

(OR)

14.b(i) Solve the following system by LU decomposition method : (8)

$$\begin{aligned} x + y + z &= 3 \\ 2x - y + 3z &= 16 \\ 3x + y - z &= -3. \end{aligned}$$

(ii) Solve the following system by Gauss-Jordan method: (8)

$$\begin{aligned} x+y+z &= 9 \\ 2x-3y+4z &= 13 \\ 3x+4y+5z &= 40. \end{aligned}$$

15.a(i) Obtain by power method the numerically largest eigenvalue and its corresponding eigenvector for the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ starting with the vector $(1,0,0)^T$ for three iterations. (8)

(ii) Construct a singular value decomposition for the matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$. (8)

(OR)

15.b(i) Using the Jacobi rotation method, find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$. (8)

(ii) Construct a QR decomposition for the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. (8)

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