Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B.E./ B.Tech (Full Time), DEGREE END SEMESTER EXAMINATION, April / May 2019

INFORMATION TECHNOLOGY

FOURTH SEMESTER (REGULATION 2012)

MA 8451 – DISCRETE MATHEMATICS

Time: 3 hours **Max. Mark : 100**

Answer ALL questions

Part – A ($10 \times 2 = 20$ marks)

1. Define Tautology with an example.
2. Define two types of quantifiers.
3. State pigeon hole principle.
4. In how many ways can six persons arranged in a line and cycles?
5. Define bipartite graph.
6. Give an example for self complementary graph.
7. Show that every cyclic group is abelian.
8. Define normal subgroup.
9. Is every poset is a lattice? Justify.
10. Show that the absorption law is valid in a Boolean algebra.

Part – B ($5 \times 16 = 80$ marks)

11. i) Define isomorphism of graphs. Are the simple graphs with the following adjacency matrices isomorphic? (8)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

ii) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G is Hamiltonian. (8)

12. (a) i) Show that $((P \vee Q) \wedge 7(7P \wedge (7Q \vee 7R))) \vee (7P \wedge 7Q) \vee (7P \wedge 7R)$ is a tautology by using equivalences. (6)

ii) Show that "It rained" is a conclusion obtained from the statements.
"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". (10)

(OR)

(b) i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(7P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

ii) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (8)

13. (a) i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2,3 or 5. (8)

ii) Solve the recurrence relation $S(n) - 7S(n-1) + 6S(n-2) = 0$, for $n \geq 2$, with $S(0) = 8, S(1) = 6,$ (8)

(OR)

(b) i) Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1} - 1}{2}$. (8)

ii) Find the generating function for the recurrence relation $S(n) - 4S(n-1) + 4S(n-2) = 2^n$, for $n \geq 2$, with $S(0) = 1, S(1) = 1$. (8)

14. (a) i) Let $\langle G, * \rangle$ be a finite group of order n and generated by an element $a \in G$. Then prove that $G = \{a, a^2, ..., a^n = e\}$ and n is the least positive integer such that $a^n = e$. (6)

ii) State and prove Lagrange's theorem on groups. (10)

(OR)

(b) i) Show that the set of permutations on the set $S = \{1, 2, 3\}$ forms a group under the right composition of permutations. (8)

ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. (a) i) Show that every chain is a lattice. Justify the converse. (8)

ii) In a distributive complemented lattice. Show that the following are equivalent.
1) $a \leq b$ 2) $a \wedge \overline{b} = 0$, 3) $\overline{a} \vee b = 1$ 4) $\overline{b} \leq \overline{a}$ (8)

(OR)

(b) i) Show that the De Morgan's laws are valid in a Boolean Algebra. (8)

ii) Let D_n denote the set of all positive divisor of the positive integer n and $|$ be a divisor relation on D_n. Show that the division relation is a partially ordered relation on D_n. (8)

***