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**B.E(FULL TIME)/ END SEMESTER EXAMINATIONS- NOV/DEC 2024**

Computer Science and Engineering-RUSA

Semester-IV

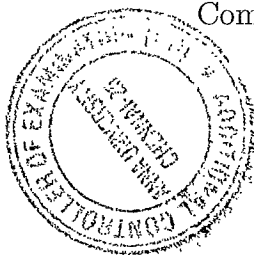
**MA6201 Linear Algebra**

(Regulation 2018-RUSA)

Time: 3 Hours

Answer ALL Questions

Max.Marks:100

**Part-A( 10 × 2 = 20 Marks )**

- 1) Show that  $W = \{(a_1, a_2, a_3); a_1 = a_3 + 2\}$  is not a subspace of  $V = R^3$ .
- 2) Examine the linear dependence or independence of the following vectors  
 $\{u_1 = (1, -1, 2), u_2 = (1, -2, 1), u_3 = (1, 1, 4)\} \in R^3$
- 3) Let  $T : R^2 \rightarrow R^2$  is linear,  $T(1, 0) = (1, 4)$  and  $T(1, 1) = (2, 5)$  then, What is  $T(2, 3)$ ?
- 4) Check whether the matrix  $A = \begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$  is Diagonalizable or not?
- 5) Define an inner product space.
- 6) Prove that  $\|x + y\| \leq \|x\| + \|y\|$ , where  $V$  is a inner product space over  $F$  and  $x, y \in V$ .
- 7) Compare Gauss elimination method and Gauss jordan method for solving the system  $Ax = B$ .
- 8) State the sufficient condition for convergence of Gauss-seidel method.
- 9) What is the use of Power method?
- 10) What are the singular values of a matrix  $A$ ?

**Part-B(8 × 8 = 64 Marks )****Answer any EIGHT questions**

- 11) Show that  $P_2(R)$ , the set of all polynomials of degree less than or equal to 2 is a vector space over  $R$  with respect to usual addition of polynomials and usual scalar multiplication.
- 12) Determine the set  $S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$  is a basis for  $R^3$  or not?
- 13) Let  $V$  and  $W$  be vector spaces, and let  $T : V \rightarrow W$  be linear. If  $V$  is finite dimensional, then Prove that  $\text{nullity}(T) + \text{rank}(T) = \dim(V)$ .
- 14) Let  $T$  be a linear operator on  $P_2(R)$  defined by  $T(f(x)) = f(x) + (x + 1)f'(x)$ . Let  $B$  be a standard ordered basis for  $P_2(R)$  then, Compute The Characteristic polynomial and Eigen value of  $T$ .
- 15) Let  $T$  be a linear operator on  $P_2(R)$  defined by  $T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$ . Test  $T$  for Diagonalizability.
- 16) Apply the Gram-Schmidt Orthogonalization process to obtain an Orthonormal basis for the subspace  $R^3$  generated by  $S = \{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$  with respect to the standard inner product.

- 17) Fit a least square line for  $(1, 2), (2, 3), (3, 5), (4, 7)$  and hence find the error.
- 18) Solve the system of equations  $x + y + z = 1, 4x + 3y - z = 6, 3x + 5y + 3z = 4$  using  $LU$  decomposition method.
- 19) Using Gauss elimination method solve  $x + 2y + z = 3, 2x + 3y + 3z = 10, 3x - y + 2z = 13$ .
- 20) Apply SOR method to solve the system of equations  $3x - y + z = 1, 3x + 6y + 2z = 0, 3x + 3y + 7z = 4$ .
- 21) Determine the Numerically largest eigen value and corresponding eigen vector for  $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$  by taking  $X_0 = (1 \ 0 \ 0)^T$  as initial vector.
- 22) Construct a Singular value decomposition for the matrix  $A = \begin{pmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix}$

**Part-C ( $2 \times 8 = 16$  Marks )**

- 23) Find the  $QR$  decomposition for the matrix  $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$
- 24) Solve the following system of equations by Gauss-Seidel method.

$$\begin{aligned} 8x - 3y + 2z &= 20 \\ 4x + 11y - z &= 33 \\ 6x + 3y + 12z &= 35. \end{aligned}$$

