

B.E(FULL TIME)/ END SEMESTER EXAMINATIONS- NOV/DEC 2024

Computer Science and Engineering-RUSA

Semester-IV

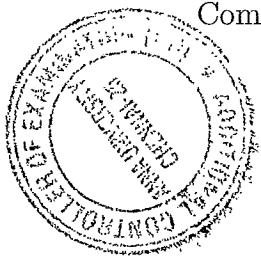
MA6201 Linear Algebra

(Regulation 2018-RUSA)

Time: 3 Hours

Answer ALL Questions

Max.Marks:100

**Part-A($10 \times 2 = 20$ Marks)**

- 1) Show that $W = \{(a_1, a_2, a_3); a_1 = a_3 + 2\}$ is not a subspace of $V = R^3$.
- 2) Examine the linear dependence or independence of the following vectors $\{u_1 = (1, -1, 2), u_2 = (1, -2, 1), u_3 = (1, 1, 4)\} \in R^3$
- 3) Let $T : R^2 \rightarrow R^2$ is linear, $T(1, 0) = (1, 4)$ and $T(1, 1) = (2, 5)$ then, What is $T(2, 3)$?
- 4) Check whether the matrix $A = \begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$ is Diagonalizable or not?
- 5) Define an inner product space.
- 6) Prove that $\|x + y\| \leq \|x\| + \|y\|$, where V is a inner product space over F and $x, y \in V$.
- 7) Compare Gauss elimination method and Gauss jordan method for solving the system $Ax = B$.
- 8) State the sufficient condition for convergence of Gauss-seidel method.
- 9) What is the use of Power method?
- 10) What are the singular values of a matrix A ?

Part-B($8 \times 8 = 64$ Marks)
Answer any EIGHT questions

- 11) Show that $P_2(R)$, the set of all polynomials of degree less than or equal to 2 is a vector space over R with respect to usual addition of polynomials and usual scalar multiplication.
- 12) Determine the set $S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis for R^3 or not?
- 13) Let V and W be vector spaces, and let $T : V \rightarrow T$ be linear. If V is finite dimensional, then Prove that $\text{nullity}(T) + \text{rank}(T) = \dim(V)$.
- 14) Let T be a linear operator on $P_2(R)$ defined by $T(f(x)) = f(x) + (x + 1)f'(x)$. Let B be a standard ordered basis for $P_2(R)$ then, Compute The Characteristic polynomial and Eigen value of T .
- 15) Let T be a linear operator on $P_2(R)$ defined by $T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$. Test T for Diagonalizability.
- 16) Apply the Gram-Schmidt Orthogonalization process to obtain an Orthonormal basis for the subspace R^3 generated by $S = \{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$ with respect to the standard inner product.

17) Fit a least square line for $(1, 2), (2, 3), (3, 5), (4, 7)$ and hence find the error.

18) Solve the system of equations $x + y + z = 1, 4x + 3y - z = 6, 3x + 5y + 3z = 4$ using LU decomposition method.

19) Using Gauss elimination method solve $x + 2y + z = 3, 2x + 3y + 3z = 10, 3x - y + 2z = 13$.

20) Apply SOR method to solve the system of equations $3x - y + z = 1, 3x + 6y + 2z = 0, 3x + 3y + 7z = 4$.

21) Determine the Numerically largest eigen value and corresponding eigen vector for $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by taking $X_0 = (1 \ 0 \ 0)^T$ as initial vector.

22) Construct a Singular value decomposition for the matrix $A = \begin{pmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix}$

Part-C($2 \times 8 = 16$ Marks)

23) Find the QR decomposition for the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

24) Solve the following system of equations by Gauss-Seidel method.

$$\begin{aligned} 8x - 3y + 2z &= 20 \\ 4x + 11y - z &= 33 \\ 6x + 3y + 12z &= 35. \end{aligned}$$

